DIRECTIONS

FOR THE USE OF

HADLEY's QUADRANT,

With REMARKS on the

Conſtruction of that INSTRUMENT.

WITH

Conſiderable CORRECTIONS and ADDITIONS.

By the Reverend Mr. *LUDLAM*,

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PREFACE.

THE inſtrument here treated of is ſo extenſive in its uſe, and ſo eaſy to manage, that one cannot but wonder it is not yet become more common. The ſeamen indeed have at laſt adopted it, and, dropping their old *Davis*'s quadrants, uſe this only for finding their Latitude; ſo that it has now got the name of the *Sea-quadrant*. The application of this inſtrument to the taking diſtances of the moon from the ſun and ſtars, in order to find the

Longitude

Longitude at ſea, begins likewiſe to be underſtood and practiſed; a buſineſs in which no other quadrant can be of the leaſt ſervice. But beſides ſeamen, who are indeed obliged to obſerve the heavens, there are many other perſons who would amuſe themſelves with the practical parts of Aſtronomy, were the neceſſary inſtruments neither expenſive nor troubleſome to uſe. This inſtrument is cheap, portable, applicable to many aſtronomical purpoſes, and of all others is the eaſieſt to manage. It is indeed its peculiar excellence that it requires no ſteadineſs of the hand to be acquired only by long practice; no fixed baſis as moſt other aſtronomical inſtruments do. Beſides aſtronomical purpoſes, it makes

makes an uſeful *Theodolite* for the ſurveying and *mapping* of counties, and far excels any other inſtrument for taking *off-ſets* in the modern way of plotting and meaſuring Land. Nothing ſeems to have hindered this inſtrument from coming into general uſe, but the want of proper directions, not only how to uſe it, but alſo to examine the goodneſs and rectify the poſition of the glaſſes; without which no obſervations made with it can be of any value. The makers themſelves are but little acquainted with the method of adjuſting the glaſſes for the back obſervation: But how well ſoever a maker may adjuſt the glaſſes at firſt, they will never keep their poſition, but muſt be

be examined and rectified from time to time; ſo that directions on this head are abſolutely neceſſary. Nothing is here directed to be done, but what has been actually executed: There will always be ſome doubt that what is propoſed upon Theory only, may not be feaſible when it comes to be tried.

The remarks are ſometimes more proper for thoſe that make than thoſe that uſe the inſtrument; poſſibly they may be of Service to both. All mathematical demonſtrations are omitted, as belonging to another place; what is here delivered not being deſigned for ſpeculation, but for practice.

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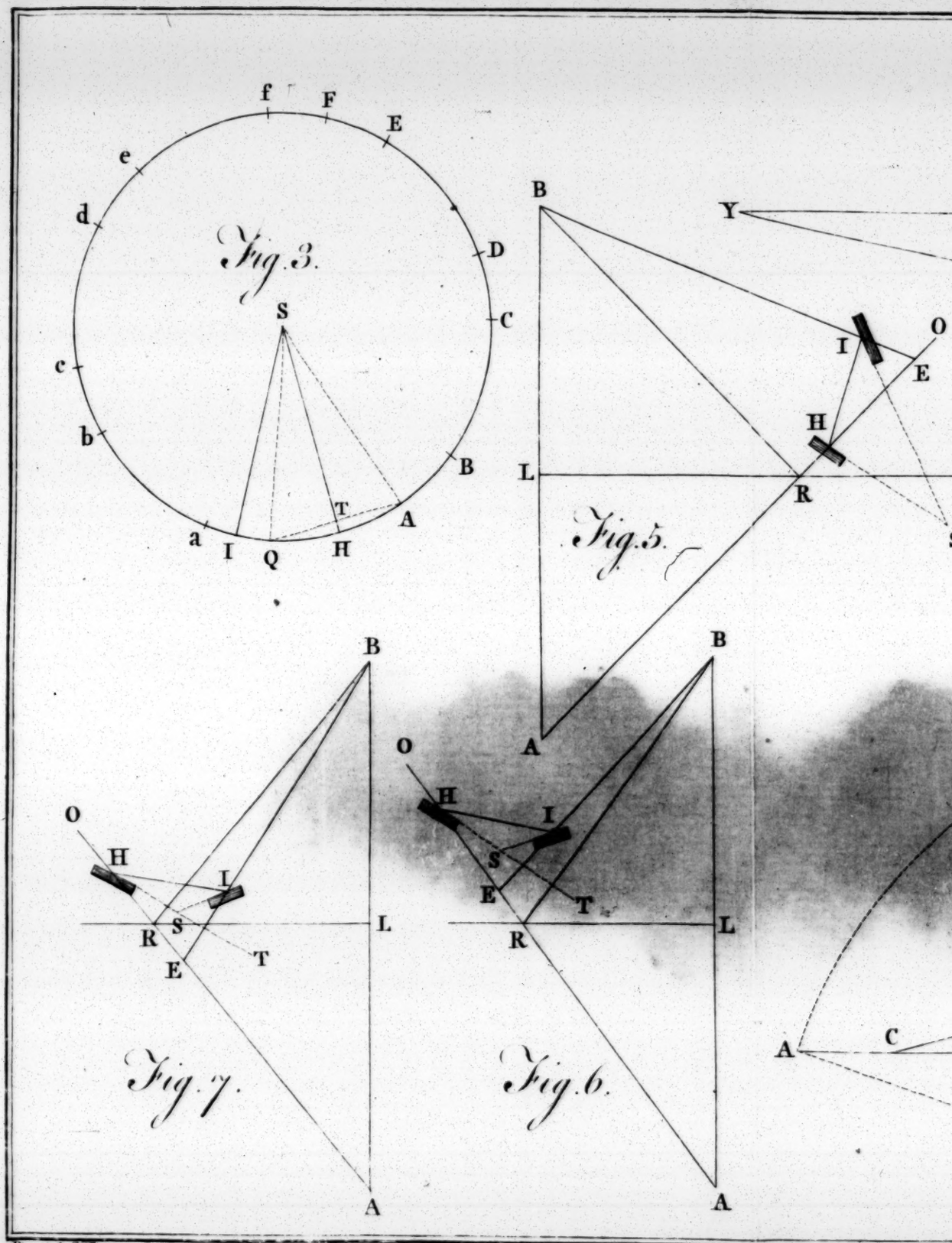
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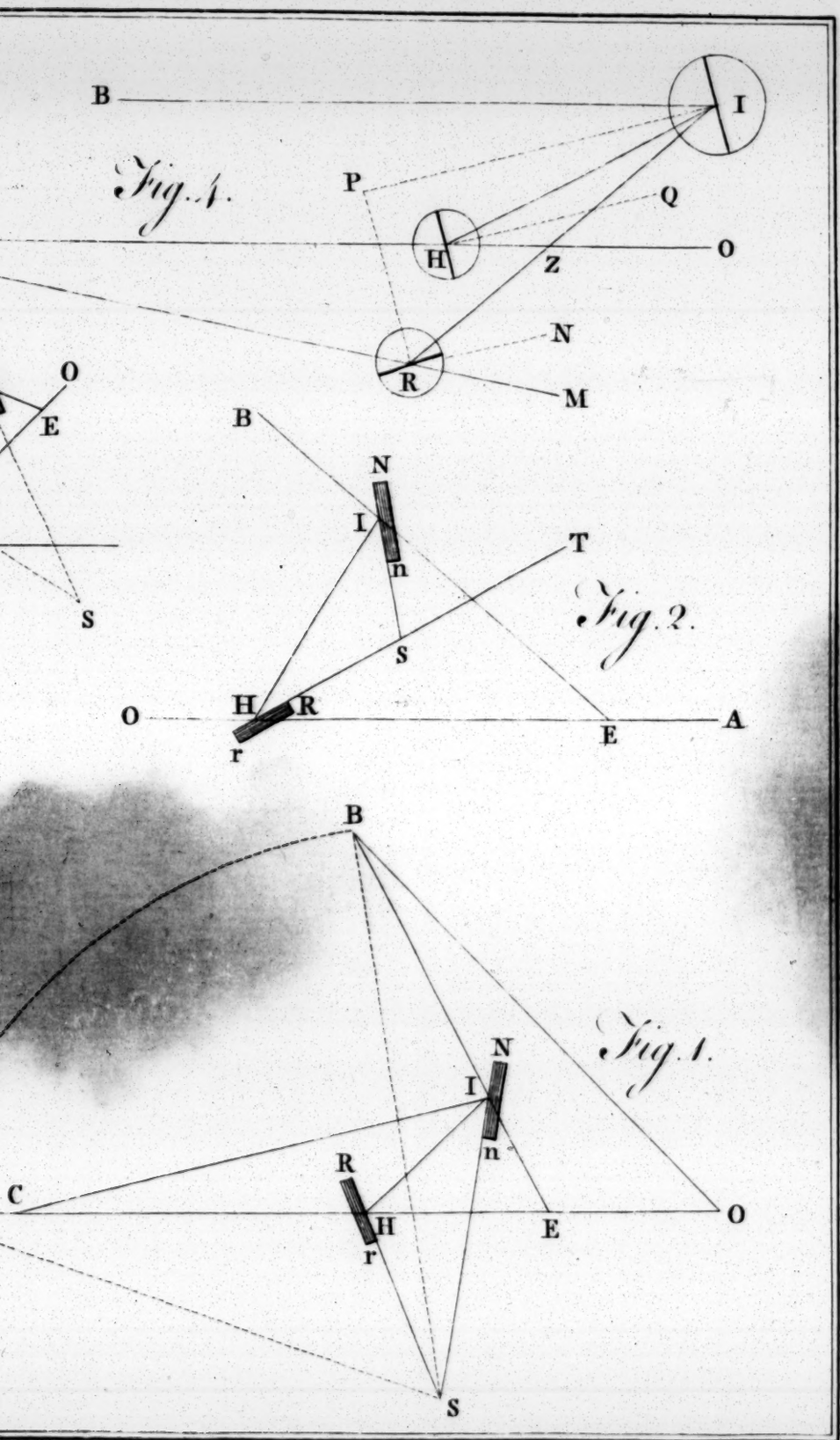

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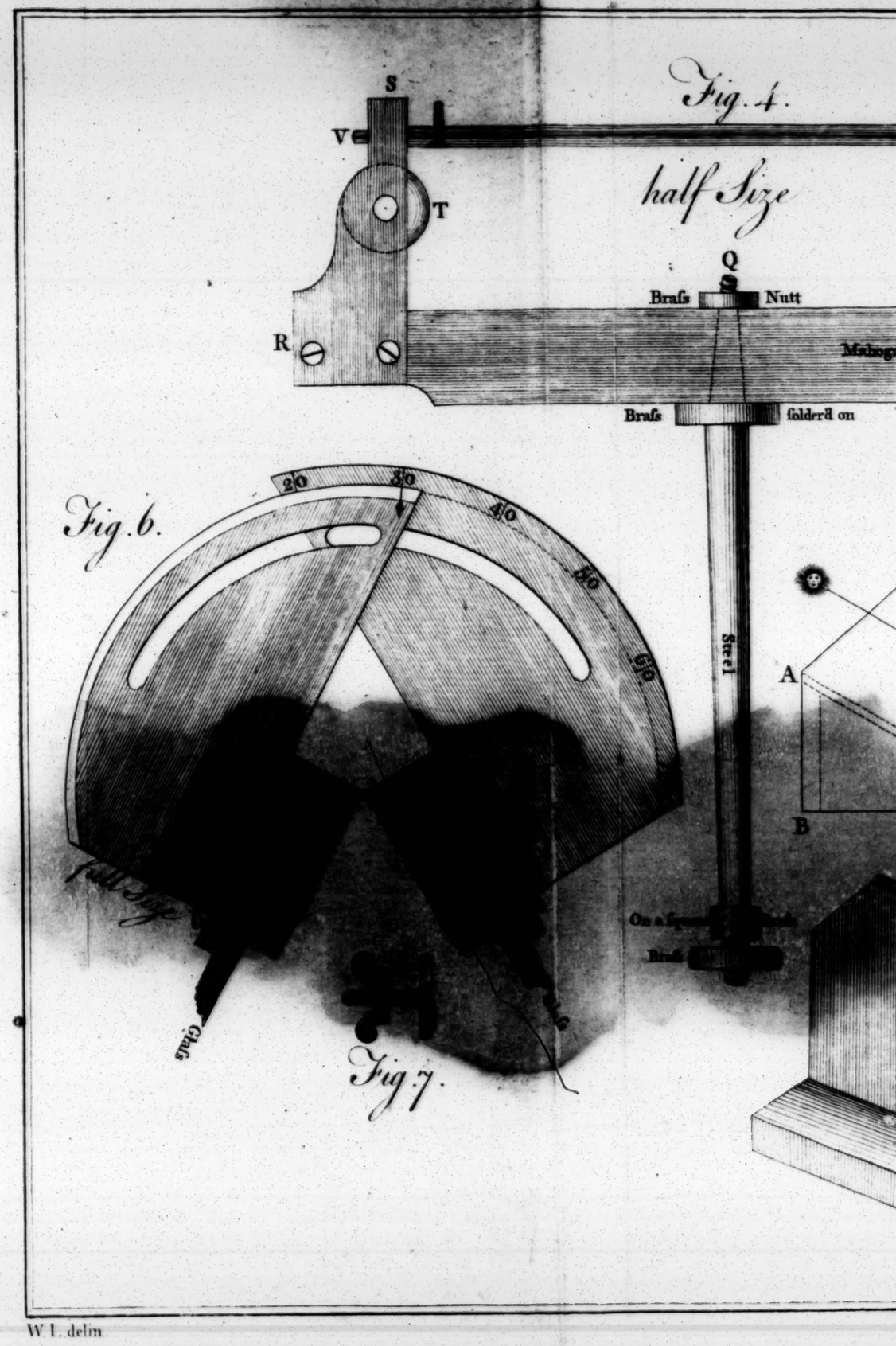
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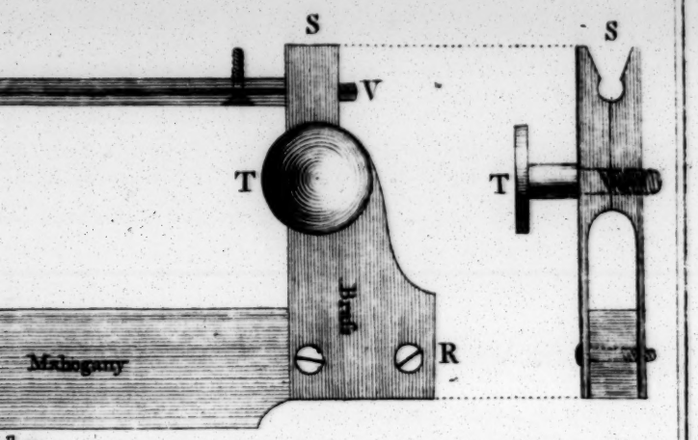

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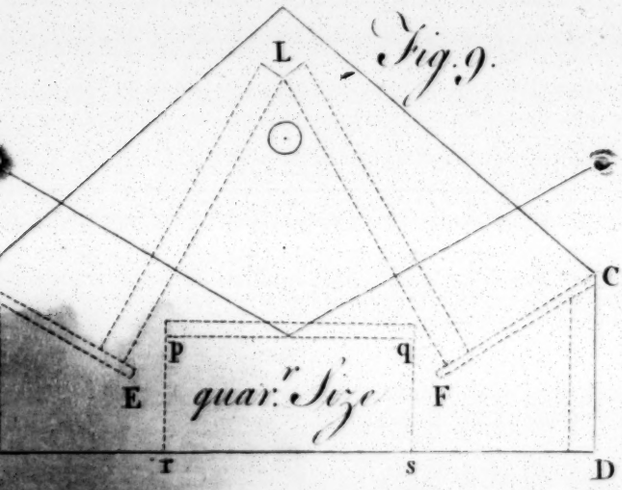

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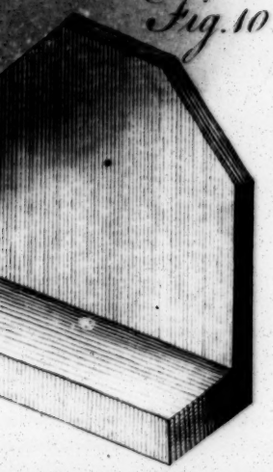

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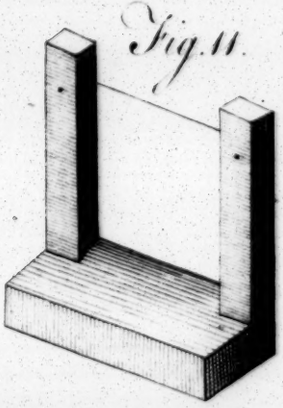

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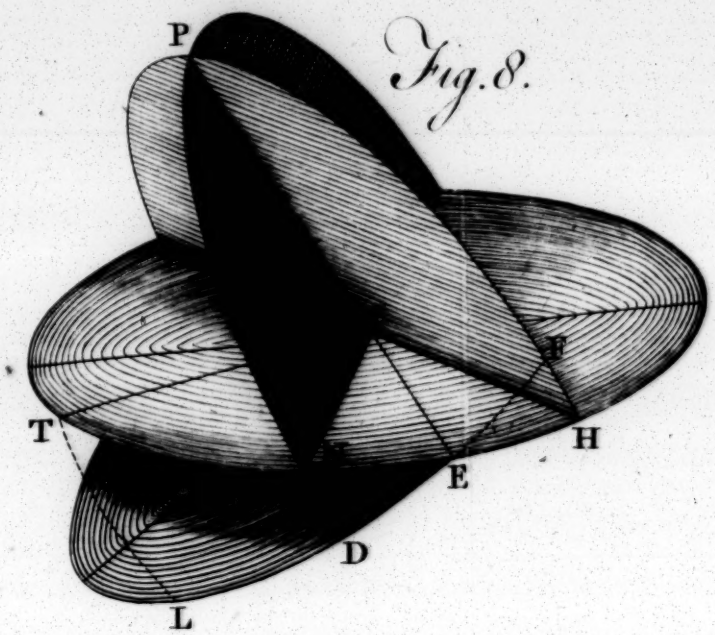

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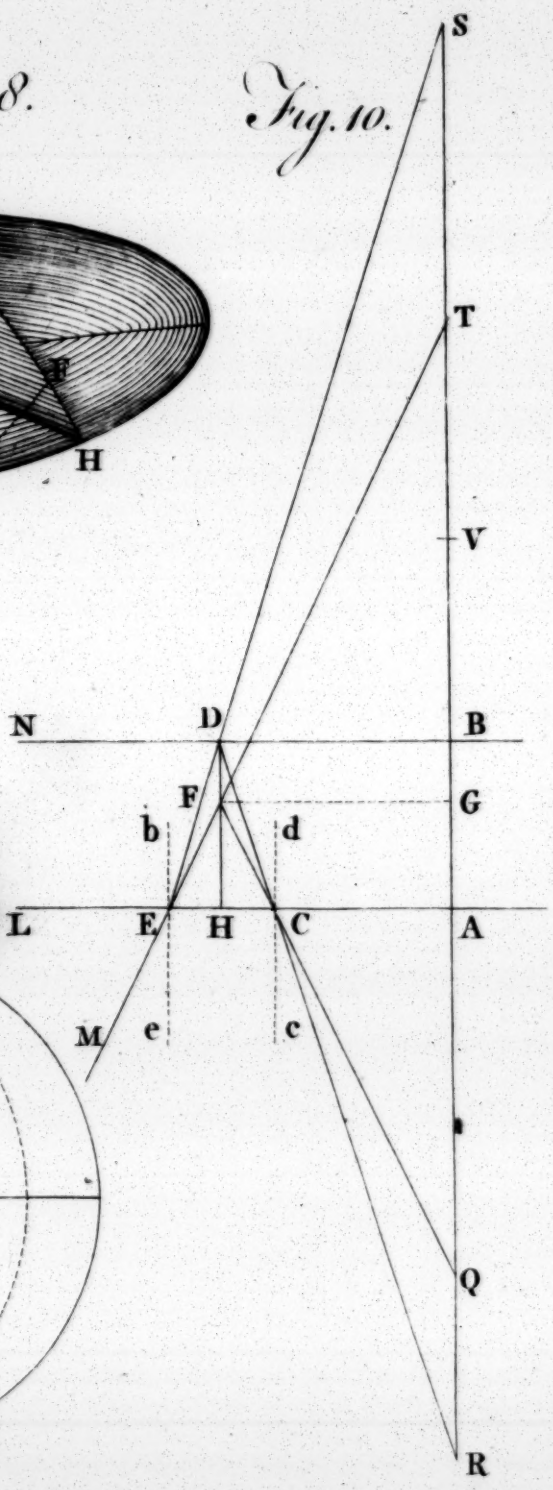

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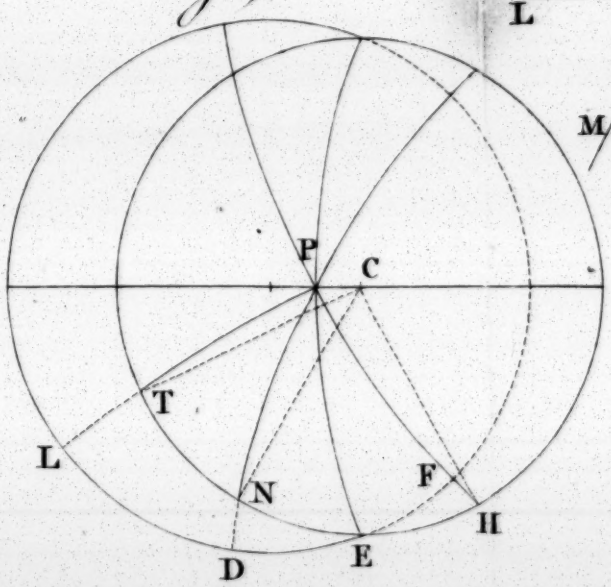

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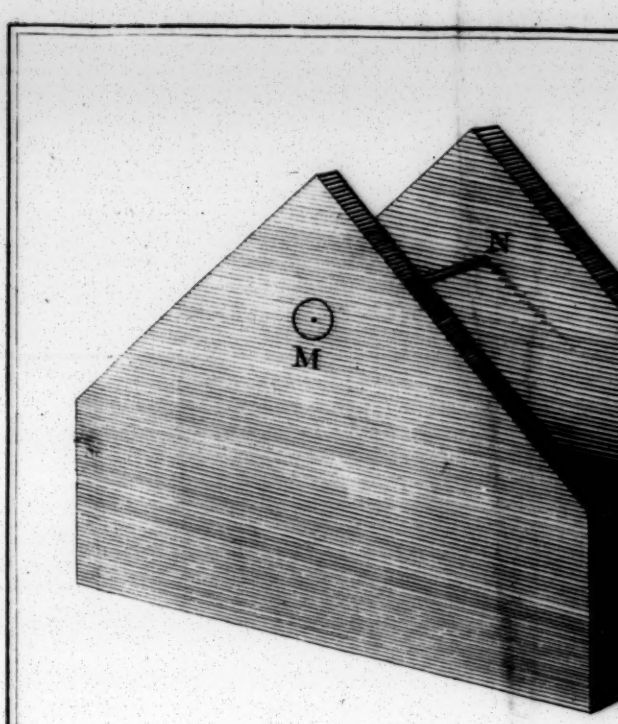

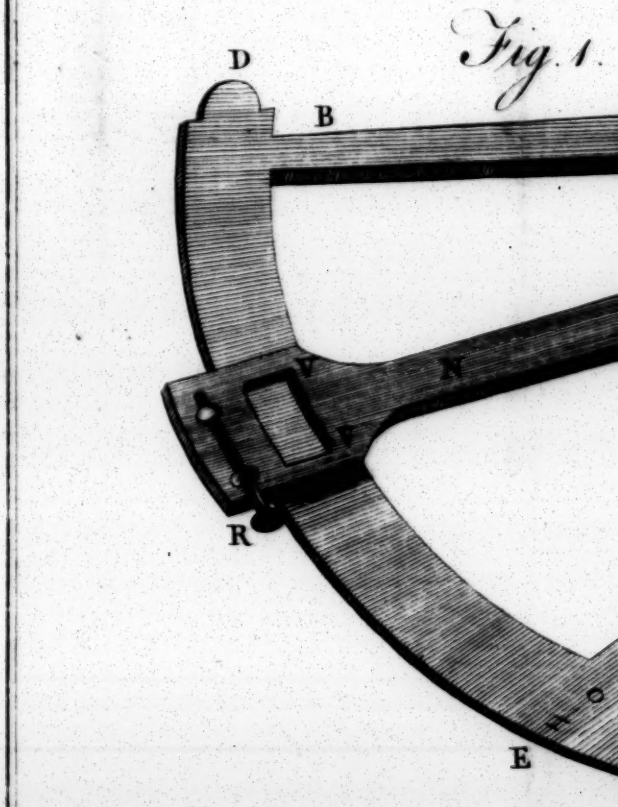

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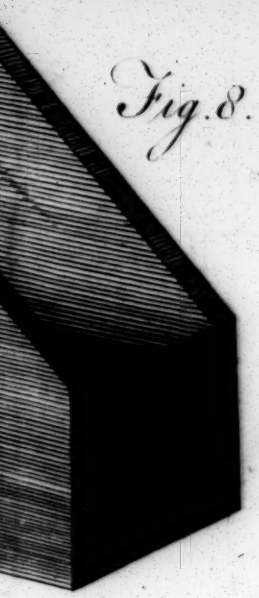

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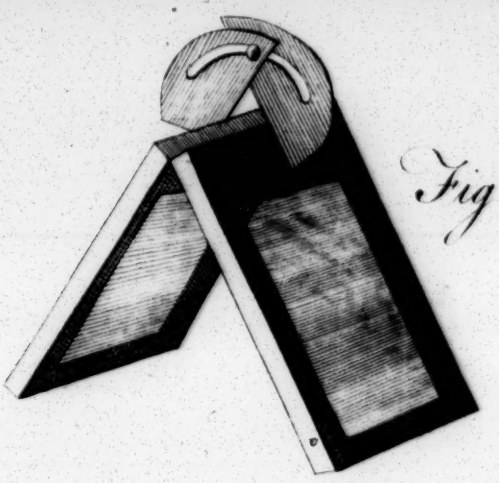

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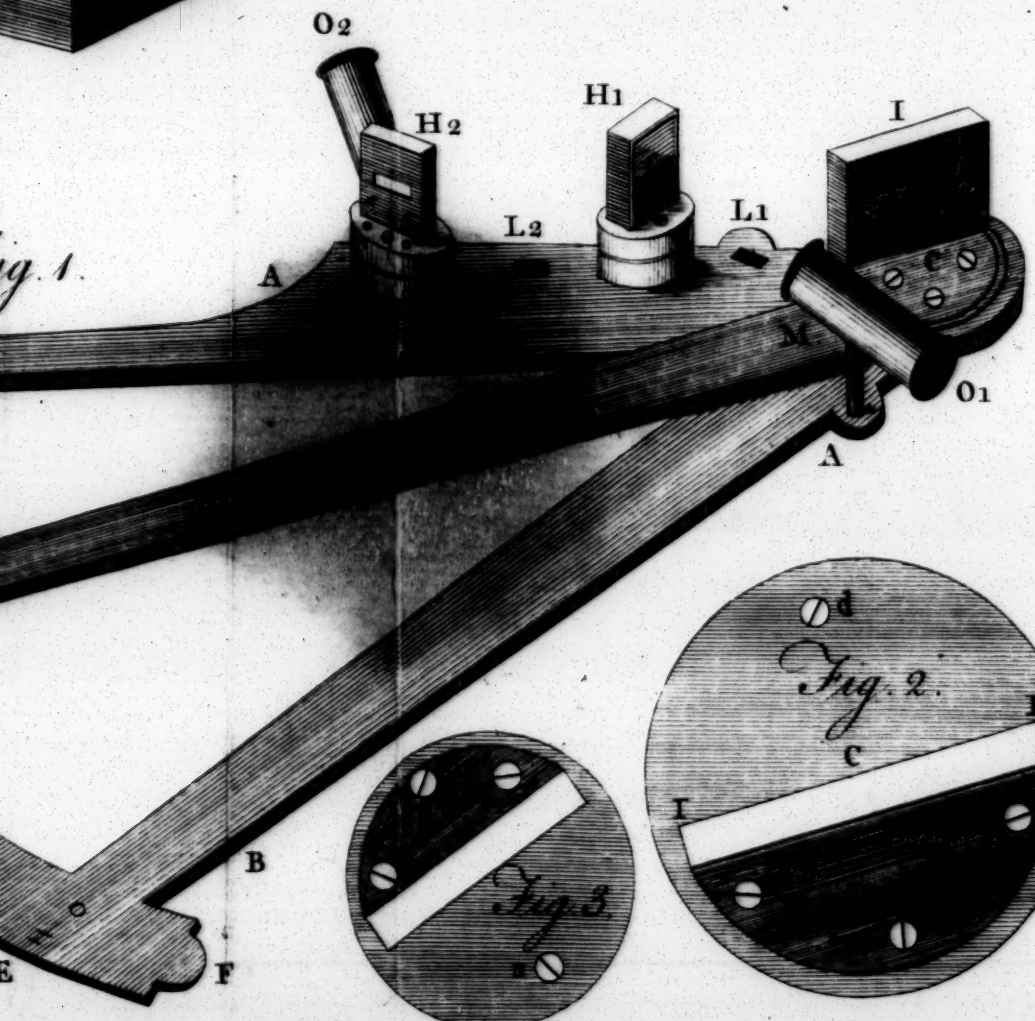

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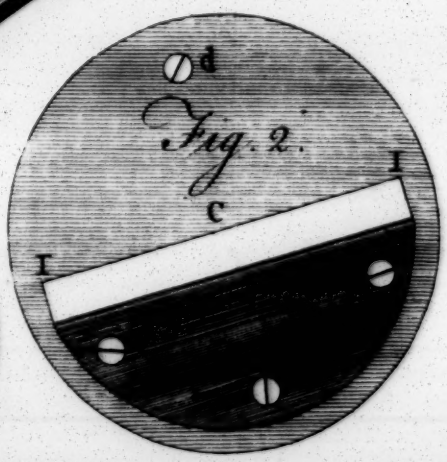

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DIRECTIONS

FOR THE USE OF

HADLEY's QUADRANT, &c.

CORRECTED and AMENDED by the AUTHOR.

A ſhort hiſtory of the invention of the reflecting inſtrument for taking angles at Sea, commonly called HADLEY's QUADRANT, *and by foreigners the* ENGLISH OCTANT.

1. THE firſt account of any inſtrument of this kind is in a paper given in to the Royal Society by *John Hadley, Eſq.* May 1731.* The author deſcribes two [illegible] octants; the latter of theſe has three ſpeculums, by [illegible]s of which any angle under 180 degrees may be taken, or the ſun's altitude obſerved at ſea either from that part of the horizon which is under the ſun, or that part which is oppoſite to it; nor does this octant differ eſſentially from thoſe now commonly made. The author likewiſe explains the principles on which theſe inſtruments were conſtructed. In a ſubſequent paper dated 1732 † Mr. Hadley ſays that an octant made of wood was produced before the ſociety when he gave in the deſcription, but that he had ſince procured another to be made of braſs; and he proceeds to give a very circumſtantial account

* Philoſ. Tranſ. Nr. 420. † Philoſ. Tranſ. Nr. 425.

of a great number of trials of them both, made on board the *Chatham* yatch, by which their great uſefulneſs at ſea was ſufficiently proved; nevertheleſs it was at leaſt twenty years before this excellent inſtrument began to come into uſe; ſo ſlow are even the beſt improvements in making their way againſt old prejudices.

2. Some years after this, viz. in 1742, a paper in Sir Iſaac Newton's hand-writing, was found among Dr. Hadley's papers after the Doctor's death. This paper contains a drawing and deſcription of an inſtrument not much different from the firſt of Hadley's; its principal properties and uſes at ſea are likewiſe there taken notice of. It ſeems therefore that in reality Sir Iſaac Newton was the *firſt* inventor of theſe reflecting quadrants, though this was not known till 1742, except perhaps to Dr. Halley, who ſeems to have taken no notice of this paper of Newton's, when Hadley's octant was publicly ſhown at the Royal Society. Mr. Hadley's great abilities, and particular ſkill in optics (of which there are many proofs in the Tranſactions) leave no room to doubt but that he likewiſe was an original inventor; and accordingly the inſtrument has always borne his name.

An Account of the ſeveral ſorts of reflecting inſtruments for taking angles.

3. There are ſeveral ſorts of theſe inſtruments for taking angles by reflection now commonly made. Large ones of braſs are apt to bend acroſs their plane unleſs made with perpendicular bars and then they are very heavy.* The common ſort are of mahogany: their diviſions are cut upon ivory, which if it does not ſhrink may do very well, as it ſhows the lines very plain and will not ruſt with the ſea air. The beſt ſize and ſort for general uſe ſeem to be thoſe of fifteen inches radius made of mahogany with a braſs plate on the limb for the diviſions.

4. Some

* Perpendicular bars are thoſe whoſe plane is perpendicular to the plane of the Quadrant, and are faſtened on the backſide of it.

4. Some of these instruments are octants, others sextants. The octants when furnished for the back observation will take any angle under 180 degrees, and answer all the purposes of a semicircle. The sextants are never furnished for the back observation and therefore take in no more than 120 degrees and that with difficulty, as the extreme reflections are very oblique and consequently the field small. They are besides more cumbersome and heavier than the octants. It would be well therefore if those who use this instrument would accustom themselves to take back as well as fore observations, and learn to examine and adjust their instrument for both purposes.* Very small instruments (of seven or eight inches radius) cannot well be fitted for the back observation, and are therefore commonly sextants. These are always made of brass, and in so small a size are strong enough without perpendicular bars. A brass octant of twelve inches radius made thick will do without perpendicular bars, the narrowness of the shape making it stronger across the plane than a sextant. An octant of this size has just room enough to admit the glasses for the back observation.

Description of a reflecting octant.

5. Figure 1 represents the octant furnished both for the fore and for the back observation. *A B* and *A B* are the two sides, *D F* is the arch or limb, *M N* is the moveable arm called sometimes the *Alidade* but commonly the index; this turns on the center *C* and carries the index glass or great mirror *I*. The end *N* of the index shows the degrees and minutes upon the limb as will be explained hereafter. *H* 1 is the horizon glass or little mirror for the fore observation. *H* 2 is the horizon glass or little mirror for the back observation. *O* 1 is the sight vane for the fore observation. *O* 2 the sight vane for the back observation. *L* 1 is the place of the red glasses for the fore observation; these are removed to *L* 2 for the back observation.

6. The

* If we may guess by the drawing, (Phil. Transf. Nr. 420) in the first octants the little speculum for the back observation was placed so near the index glass, that an altitude of the sun (unless very high) could not be taken, the head of the observer being in the way. Seamen finding the back observation frequently impracticable in the first instruments, would naturally be prejudiced against it in all others; which I believe is the chief reason why the back observation has been so much neglected.

6. The index glaſs *I* is put into a braſs frame and is drawn tight againſt an upright plane of braſs by two ſcrews *e* and *f* behind the frame.* To the bottom of this upright plane is joined another braſs plane at right angles to it, by which the whole braſs work carrying the index glaſs is ſcrewed down to the index.

7. This braſs work is ſcrewed to the index by three ſcrews behind the glaſs at *C* fig. 1. and again repreſented in fig 2. Two of theſe *a* and *b* draw the work down to the index, the third *c* puſhes againſt the other two. Theſe ſcrews ſerve not only to faſten the braſs work but alſo to incline the index glaſs backwards or forwards in order to adjuſt it.

8. Sometimes this braſs work is ſcrewed faſt to a circular plate of braſs which reſts upon two points placed underneath the ends of the index glaſs. The ſcrews *c* and *d* (fig. 2) faſten this circular plate down upon thoſe points, and by drawing againſt each other, incline that plate and the glaſs upon it backwards and forwards in order to adjuſt it as before.

9. The fore horizon glaſs is likewiſe put into a braſs frame and kept tight by two ſmall ſcrews behind the frame, like the index glaſs.

10. The braſs work containing the horizon glaſs is ſcrewed faſt to a circular plate of braſs by two ſcrews *h h* fig. 3 behind the glaſs. This circular plate reſts upon two points in another circular plate underneath the former; theſe points are placed under the ends of the horizon glaſs.

11. There are two adjuſting ſcrews *s s*, one before and one behind the glaſs, which draw the circular plate down upon the points. Theſe adjuſting ſcrews by drawing againſt each other, ſerve to incline the glaſs backwards and forwards in order to adjuſt it.

12. Beſides

* The index glaſs is often diſtorted by the preſſure of theſe ſcrews behind. To prevent this, in ſome inſtruments, the upright plane of braſs againſt which the back of the glaſs lies, has three points, and the braſs frame that bears againſt the front of the glaſs, has likewiſe three points directly oppoſite to the former. On the back of the frame is a ſpring with three feet, anſwering to thoſe points. When that ſpring is ſcrewed on the glaſs is then preſſed between thoſe points, in three places only, and therefore it cannot be diſtorted, for a plane will always conform its poſition to any three points.

As the glaſs is preſſed by the force of the ſpring only, and not by the whole force which the hand can give to the ſcrew, the glaſs cannot be broken by ſcrewing up; nor is the ſcrew ſo liable to looſen and return back, as when it preſſes directly againſt the glaſs.

12. Beſides the adjuſting ſcrews *s s*, the horizon glaſs has likewiſe an adjuſting lever by which it may be turned circularly round an axis perpendicular to the plane of the octant. This lever is uſually on the backſide of the octant, and has a contrivance to turn it ſlowly, and a button-ſcrew to fix it. In ſome inſtruments this glaſs is turned about by an endleſs ſcrew.

13. The lower half only of the fore horizon glaſs is ſilvered over, the upper half is left clear, through which, in making obſervations, one of the objects is ſeen directly, while the other is ſeen by reflection from the lower half of the glaſs.* It was the cuſtom formerly to ſilver this glaſs at the top and bottom, and leave a tranſparent ſlit in the middle; but this is now left off.

14. The back horizon glaſs is fixed in a frame in the ſame manner and has the ſame adjuſting ſcrews, and adjuſting lever as the fore horizon glaſs. It differs only in this, that it is ſilvered both at the top and bottom, and has a tranſparent ſlit acroſs the middle, through which one of the objects is ſeen directly while the other is ſeen by reflection. †

15. The foreſight vane is made in various ways. Generally it is only a flat plate of braſs with two holes in it. One hole is exactly over againſt the ſeparation between the ſilvered and tranſparent part of the horizon glaſs; the other hole is above the former and oppoſite to the middle of the tranſparent part of the horizon glaſs. The lower hole is to be uſed in common caſes; the upper hole when the object is ſo bright as to be ſeen by reflection from the unſilvered part of the glaſs. There are little ſhutters to cover the hole not in uſe.

16. Inſtead

* If the octant was to be uſed for land objects only, it would be better if the upper half of the horizon glaſs was cut off. For though land objects may be ſeen through it, yet they appear much darker. The great uſe of this part of the horizon glaſs is in obſervations of the ſun, whoſe image is bright enough to be ſeen by reflection from this part though not ſilvered; and thus that reflected image can be made to coincide exactly with other objects ſeen directly through the very ſame point of the glaſs.

† I ſuppoſe the reaſon of thus leaving only a ſlit tranſparent, to be as follows. The eye (when a plain hole is uſed for the ſight) being very near the glaſs, a narrow ſlit is ſufficient to give a large field for objects ſeen by direct viſion; the remaining part of the glaſs both above and below is ſilvered, that the objects to be ſeen by reflection may be more eaſily found. This conſtruction however is inconvenient, eſpecially for land objects when either a plain tube or teleſcope is uſed. This horizon glaſs ſhould undoubtedly be ſilvered juſt in the ſame manner as the other.

16. Inſtead of the vane with holes, a plain tube is frequently uſed to direct the ſight, and ſometimes a teleſcope. When a plain tube is uſed, its height ſhould be changeable ſo that its axis may be made to point either againſt the ſeparation between the ſilvered and tranſparent part of the horizon glaſs, or elſe againſt the middle of the tranſparent part, which is neceſſary in taking the altitude of the ſun at ſea. The teleſcope is always made to riſe and fall above and below the ſeparation of the ſilvered and tranſparent parts of the horizon glaſs, ſo as to take in more or leſs of the one or the other. By this means either object (that is ſeen directly or that by reflection) may be made to appear through the teleſcope more or leſs bright as circumſtances may require. This teleſcope is likewiſe furniſhed with ſmoaked glaſſes in a braſs frame which ſlides acroſs the eye hole. This braſs frame contains two glaſſes whoſe inward faces are ſmoaked with the flame of a candle; one end lightly, and then gradually darker and darker to the other end. A frame of braſs is interpoſed between the ſmoaked glaſſes to prevent their touching each other ſo as to rub off the ſoot. Theſe ſmoaked glaſſes are to be uſed in viewing the ſun when both its direct and reflected images are brought into the teleſcope together.

Some inſtruments are furniſhed with all three kinds of ſights, plain holes, a tube, and a teleſcope, to be put on and uſed in each others place.

17. The back vane does not differ in its conſtruction from the fore vane. The tube or teleſcope is commonly made to fit both vanes. When there is no teleſcope there ought to be a director of the ſight for the back obſervation as recommended by *Hadley*. This was very injudiciouſly laid aſide, but is lately brought into uſe again. The director is a frame of braſs *T T* to be fixed occaſionally on the ſide of the limb *A B* where the diviſions begin. This frame has a wire ſtretched acroſs it, ſo as to be parallel to the plane of the octant and at the ſame height above it with the ſight *O 2*. Its uſe is to direct the ſight parallel to the plane of the octant.*

* It may be remarked that in common octants the ſight vanes are made with holes only, and thoſe very ſmall. Being chiefly deſigned for taking the ſun's altitude at ſea, the brightneſs of the object makes it eaſy to find the reflected image in the horizon glaſs, and ſmall holes are ſufficient to ſhew it; but for land objects a ſhort tube is very neceſſary to take off the light of the ſky, which entering ſideways through a plain hole almoſt effaces the light coming from the object.

18. The two objects which coincide are seen, the one by a ray passing through, the other by a ray reflected from the same point of the horizon glass to the eye; this ray is called the *Visual ray*. When this ray is considered merely as a line drawn from the middle of the horizon glass to the eye-hole of the sight vane, it is then called the AXIS OF VISION. The axis of the telescope or tube used to direct the sight, coincides with, and is also considered as the *Axis of Vision*.

19. The red glasses, (or as some call them, dark glasses,) are round glasses put into square frames of brass. These glasses are not made fast in their frames but may be turned about to adjust as will be shown hereafter. Their use is to make the object seen by reflection appear of a dark red colour, when it would otherwise be too bright. One of these glasses is of a lighter, the other of a darker colour; the former is to be used in taking distances of the moon from the stars in order to find the longitude at sea, the latter in observations of the sun. If the sun is very bright both red glasses may be used together. These glasses turn out of the way in other observations.

20. The limb *D F* is divided into degrees and half degrees, every ten degrees of which is numbered, beginning from 0 at *E* and going towards the left hand to 94 or 95 degrees. This part of the limb is called the *quadrantal arch* or *internal arch*. The part of the limb *E F* lying the contrary way from 0 is likewise divided into degrees and half degrees and is numbered from 0 at *E* the contrary way towards the right hand. This part *E F* is called the *arch of excess* or *external arch*. The odd minutes are shown by a scale of divisions *V V* on the index called sometimes a *Nonius* and sometimes a *Vernier*.*

21. The middle line of this scale of divisions or Vernier, is distinguished by being longer than the rest; against this line is set 0. The other divisions are numbered, every five, from 0 to 15 on the right hand, and then beginning from the extreme division on the left hand from 15 to 0 thus:

15 20 25 0 5 10 15

22. The place of the middle line of the Vernier among the divisions on the limb, shows the intire degrees and odd half degree

* This invention has been commonly attributed to *Petrus Nonius*, (or *Pedro Nunez*) and it has accordingly borne his name; but it was in reality the invention of *Pierre Vernier* a Frenchman in 1631; and of late both French and English writers have called it by his name.

degree (if any) ſubtended by the objects obſerved. The odd minutes (always under 30) are ſhown by the Vernier in manner following. Look among the ſtrokes on the Vernier for one that ſtands directly oppoſite to ſome ſtroke among the diviſions on the limb; that ſtroke numbered among the diviſions of the Vernier ſhows the number of minutes to be added to the intire degrees and half degree (if any) ſhown as aforeſaid on the limb.

23. If the angle ſubtended by the objects is taken upon the arch of exceſs, ſo that the middle line of the Vernier falls between *E* and *F*, then in finding the odd minutes by the Vernier, the numbers on the ſcale are to be read in a contrary order, juſt as if they had been figured thus,

15 10 5 0 25 20 15:

or which is the ſame thing, you may firſt read off the odd minutes in the uſual way, and then their complement to 30 will be the real number of minutes to be added to the degrees and half degree ſhown on the arch of exceſs. See alſo par 27.

24. What has been ſaid relates intirely to the fore obſervation; in the back obſervation, the angle ſubtended by the two objects is the *ſupplement to* 180 *degrees* of that ſhown on the limb, and eſtimated by the figures engraved thereon, as aforeſaid.

25. In taking the ſun's altitude at ſea by the back obſervation, the two objects whoſe angle is determined are the ſun, and that part of the horizon which is oppoſite to the ſun: but the ſupplement of this angle, is that which is contained between the ſun and the part of the horizon under the ſun; therefore in ſuch back obſervations at ſea, the numbers ſhown on the limb immediately give the ſun's altitude, juſt as in the fore obſervation. —— In other caſes of the back obſervation the angle between the objects is had by ſubtracting the degrees and minutes ſhown (and eſtimated by the figures) on the limb, from 180 degrees as was ſaid before.*

26. The

* To make the inſtrument have the perfect ſemblance of a ſemicircle, the numbers on the limb ſhould be returned back again from *D* to *E*, that is from 90 to 180 degrees. There ſhould likewiſe be another ſet of numbers on the Vernier in a contrary order; but as ſo many ſets of figures might create confuſion, it is better to have one ſet of figures only as uſual; to read off the degrees and minutes in the common way, and take their ſupplement to 180 degrees in the back obſervation.

26. The index has a ſetting ſcrew on the backſide to fix it; nevertheleſs after the index is thus ſet, it may yet be moved ſlowly backwards or forwards by the regulating ſcrew *R*, fig. 1, by which means the two objects may be nicely brought into contact. Some inſtruments have only a ſetting ſcrew, and no regulating ſcrew, but this is a defect.

27. In ſome inſtruments the Vernier is numbered in a manner different from that we have deſcribed. The figure 0 ſtands againſt the extreme diviſion at one end, commonly on right hand, and the numbers run regularly on to the other end, thus:

30 25 20 15 10 5 0;

but whenever the angle is taken on the arch of exceſs, theſe numbers muſt be read in a contrary order, thus:

0 5 10 15 20 25 30; as was before directed*.

28. The degrees on the limb are likewiſe differently ſubdivided in different inſtruments; this will occaſion a difference in the ſcale of the Fernier. When the degrees are ſubdivided into three parts, each part contains 20 minutes, and the numbers on the Vernier run only to 20 not to 30; thus; 10 15 0 5 10; or thus; 20 15 10 5 0. All the forgoing rules hold in this caſe. For inſtance, if the angle be taken on the arch of exceſs thoſe figures muſt be read thus;

10 5 0 15 10

or in the latter caſe thus;

0 5 10 15 20

Otherwiſe the numbers may be read juſt as they are engraved on the Vernier, and their complement to 20 taken for the true number of minutes to be added to the odd 20, or the odd 40 ſhown on the limb.

29. In ſome inſtruments the index is ſo made that the ſcale of the Vernier is in the ſame plane with the ſcale of degrees

* The 0 may be made to ſtand at either end, and the numbers to read either in the ſame order with the figures on the limb, or in the contrary order. The length of the intire ſcale of the Vernier muſt be equal to as many of the leaſt parts into which the limb is *actually* divided, as are contained in the Vernier and *one more* in the latter caſe, but *one leſs* in the former. Thus where the limb is actually divided into half degrees, and the Vernier of courſe has 30 parts to diſtinguiſh each half degree into minutes; the length of thoſe 30 parts of the Vernier, muſt be equal to 31 half degrees on the limb when the figures on the Vernier are to be in an order *contrary* to thoſe on the limb; and to 29 half degrees when the figures on the Vernier are to be in the *ſame* order with thoſe on the limb.

on the limb; in others the ſcale of the Vernier comes down to the limb with a ſlope or feathered edge. The coincidence of the ſtrokes may be more exactly ſeen when both ſcales are in one plane, provided the limb be wrought ſo true that the edge of the Vernier makes a perfect joint with the limb in every part: But as this is ſcarce poſſible, the method of making the Vernier with a feathered edge ſeems preferable.

In ſome inſtruments the index moves underneath the plane that is on the back ſide. This is very inconvenient; for the inſtrument can never be laid down without danger of diſturbing the index; indeed good inſtruments are ſeldom made in this form.

30. It is uſual to varniſh all the braſs work of a bright gold colour; but it would be better if the frames that ſurround the glaſſes, and the parts on which they ſtand were varniſhed with a dead black. The glare of the braſs-work in the former caſe gives a falſe light which enters the eye along with the light from the objects and in a manner overcomes it.

Of a Support for the Octant.

31. Although that degree of ſteadineſs which is neceſſary in all other inſtruments is not required in this, yet a proper ſtand to ſupport to ſupport the weight of it is very convenient. One of this ſort is deſcribed in the *Aſtronomical Obſervations made at Cambridge, &c**. That ſtand is particularly deſigned for taking diſtances of the moon from the ſun or ſtars, or the diſtances of ſtars from each other. A far more ſimple apparatus will ſerve when the octant is to be uſed only to take angles between land objects in making ſurveys. The part marked *QRST* in plate firſt of that work (and here in figure 4) is alone ſufficient for this purpoſe. The axis *QQ* may be put through the ſocket of the three-legged ſtaff made to ſupport a common *Theodolite* in the very place of the long ſcrew that ſcrews the *Theodolite* down upon the ſocket. This apparatus may be taken away and the *Theodolite* put upon the ſocket occaſionally.

32. Fig. 4. is the profile of an apparatus of this ſort made for a twelve inch octant. The axis *V V* on which the

* Printed for T. Cadell, London.

octant

octant ſwings is one entire piece of braſs, and is ſcrewed on the back ſide of the octant. It ſhould be ſo placed as to paſs through the center of gravity of the whole inſtrument and be parallel to the axis of viſion belonging to the fore horizon glaſs.

Care ſhould be taken that the ſupports or braſs heads *S S* do not lie in the direction of the axis of the back horizon glaſs. For then theſe heads will intercept the direct view through the tranſparent part of that glaſs when the face of the octant is downwards: * for the octant may be placed on ſuch a ſupport with its face either upwards or downwards, and thus the fainteſt of the two objects be always ſeen directly and the brighteſt by reflection.

A ſmall addition to this work will make it a uſeful ſtand for the octant when applied to the taking altitudes of the ſun or ſtars by reflection from the ſurface of a fluid.

Of the apparatus for taking altitudes by reflection from the ſurface of a fluid.

33. It is neceſſary to ſkreen the ſurface of the fluid from the action of the wind by two panes of glaſs ſet over the fluid ſo as to be nearly perpendicular to the rays tranſmitted through them. Theſe planes are each three inches broad and 4 inches long, and are pinned in mahogany frames. The ſides and bottom of the frames are half an inch wide, the top one inch wide. The frames are joined together by hinges ſcrewed to the top. To the top of each frame is likewiſe ſcrewed a braſs arch, by which they can be ſet to any inclination between 180 and 50 degrees. The figures on the arch ſhow half the inclination of the planes; therefore the planes muſt be ſet ſo that the index may point on the arch to the expected latitude. The perſpective view of theſe planes is given in Fig. 5. and the plan of the braſs arches in

* It is to avoid this that the axis *V V* is made parallel to the axis of viſion of the fore, and not of the back horizon glaſs, not in an intermediate direction. Another reaſon is, that notwithſtanding the axis *V V* is not parallel to the axis of viſion of the back horizon glaſs, yet if the octant be made to vibrate about the axis *V V* the reflected image of any object ſeen in the back horizon glaſs will deſcribe an arch, the convex ſide of which is turned towards thoſe objects whoſe angular diſtance from the given object exceeds that of its image. The advantage of this will be better underſtood hereafter.

An octant of leſs than 12 inches radius, cannot well be mounted in this manner, becauſe its center of gravity will fall too high, and the axis *V V* paſs too near the ſight vanes.

Fig. 6. * Theſe braſs arches ſlip beſide each other and have each a long circular ſlit, through which paſſes a ſetting ſcrew. The profile of this ſcrew is ſeen in Fig. 7. On the inſide of the head of the ſcrew is a ſteady pin which goes through the ſlit of one of the arches and keeps the head from turning round. A ſmall part *a b* of each arch returns at right angles to the plane of the other part, by which it is ſcrewed to the mahogany frame.

34. Fig. 8 is the perſpective view of the reflecting box, and Fig. 9 is the profile to one fourth of the real ſize; the ſections of the planes are expreſſed by dotted lines but without their braſs arches; theſe ſections are marked by *L E* and *L F*, in fig. 9. The reflecting box is dove-tailed together at the corners *A B* and *C D*, but open every where elſe and without a bottom. It is four inches wide, that is juſt wide enough to admit the planes which reſt upon the thin boards *A E* and *C F*. Theſe boards ſlide into grooves made in the ſides of the box. The braſs pin *M N* fig. 8 keeps the top of the box from warping or flying out. The water trough *p q r s* is made of pewter; four inches long, three inches wide and two inches deep, and is blacked within. It has a cover which ſkreens about a quarter of an inch of the water next the ſides of the trough all round, to hide the irregular reflections made by air-bubbles which adhere to the ſides. There is another trough of wood of the ſame dimenſions to be uſed in the place of the former: this has a falſe bottom within half an inch of the top to hold quickſilver. When the altitude of the ſtar is very great, the trough may then be covered with one of the planes laid flat upon it; the mahogany frame being made to fit the top of the trough for that purpoſe.

35. The two ſurfaces of each glaſs plane ſhould be perfectly parallel, though a ſmall variation is of no conſequence; partly becauſe the rays paſs almoſt perpendicularly through them, and partly becauſe by turning the planes, their refractions will have contrary effects and the errors will balance each other, as will be ſhown hereafter. The parralleliſm of theſe glaſs planes may be examined by the methods deſcribed in the following paragraphs.

* Figures 2 3 4 6 7 9 are profiles or ſections laid down from the original by ſcale and compaſs. Fig. 1 5 8 10 11 are perſpective views; their principal lines were marked out mechanically, with an inſtrument by which you may draw the perſpective view of any object with eaſe, expedition and accuracy.

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The METHOD of examining the GLASSES belonging to the OCTANT.

All the glaſſes belonging to this inſtrument ſhould have their two ſurfaces perfect planes and exactly parallel to each other.

36. To examine whether the two ſurfaces of any of the reflecting glaſſes are exactly parallel, obſerve the image of ſome diſtant object reflected very obliquely from the glaſs;* if that image appears ſingle it is a ſign that the two ſurfaces are parallel. † On the contrary, if the edges of the object thus ſeen by reflection appear to be ſeparated ſo as to make two images, the ſurfaces of the reflecting glaſs are then inclined each other.* This examination will be more perfect if the reflected image be viewed through a teleſcope magnifying eight or ten times. If the image then appears ſingle, its parts ſharp and well defined, it is a ſign that the reflecting glaſs is a good one indeed.

37. It may be proper to take notice of the reflected image of the ſun or moon when the index ſtands at 90 degrees, the reflection from the index glaſs being then the moſt oblique poſſible. If the ſun's image then appears ſingle, it is a ſign the glaſſes are good.

38. But the ſurfaces of the reflecting glaſſes ſhould not only be parallel but likewiſe perfect planes. † To examine ths,

* By diſtant objects we ſhall for the future underſtand *firſt* and *principally* any object in the heavens, the ſun, moon, or ſtars. *Secondly* the ſea horizon. *Thirdly* land objects not leſs diſtant than a quarter of a mile; bnt the more diſtant theſe are, the better.

† For the diſtance between the two images reflected by the two ſurfaces (when parallel) is twice the thickneſs of the glaſs; neglecting the refractions between the ſurfaces which leſſen the diſtance between thoſe images. If the object be ſo far off that this diſtance ſubtends an angle leſs than a minute, and is conſequently inviſible, the two images will then appear as one.

* If the ſurfaces either of the index glaſs or the horizon glaſs ſhould not be exactly parallel, but, when produced, interſect each other; yet if theſe glaſſes are ſo placed that this interſection is parallel to the plane of the inſtrument it will not occaſion any error in taking angles. For though there will be two images formed, yet theſe two images will be one below the other with reſpect to the plane of the octant, and therefore each at the ſame angular diſtance from the object itſelf.

† The reflecting glaſſes are often diſtorted by the ſcrews which draw the braſs frame

this, look out for two distant objects: move the index till the reflected image of one object coincides with the other object seen directly; making them unite just at the separation between the silvered and transparent part of the horizon glass. Turn now the octant about *in its own plane*, so as to make the united images move along the line of separation in the horizon glass. If the images continue united, then the reflecting planes are flat; but if the image of one object either passes beyond or recedes from the other in the direction of the line of separation in the horizon glass, (that is in the direction of the plane of the octant) the glasses are faulty. Any separation of the images in a direction perpendicular to the plane of the octant is of no consequence.

39. In this examination care must be taken to hold the octant exactly in a plane passing through the eye of the observer and the two objects, as will be more particularly insisted upon hereafter. If either of the objects are bright enough to be seen by reflection from the transparent parts of the horizon glass, its image may be brought to the upper part of the horizon glass by holding the octant out of the proper plane. The other object may likewise be seen at the same time through the upper part of the horizon glass: but though they were united before in the middle of the glass, they will not be united now in this upper part, not owing to any fault in the glasses but to the wrong position in which the octant is held, the ray by which the objects are seen, not being parellel to the plane of the octant. When the sight vane has two holes, it may be proper to try whether the objects appear united when seen through each hole, in the two parts of the horizon glass opposite to those holes. It requires experience in the management of the instrument to make this examination properly, so as not to mistake that for a fault in the glasses which is only a fault in the manner of holding the octant. See par. 108.

Of the Red Glasses.

40. The examination of these glasses must be made when the sun is on or near the meridian. Hold the octant nearly

frame tight against the glass and by that means press the glass against the upright plane, See par. 6 and 9. Sometimes the face of this plane, sometimes the glasses themselves *cross-wind*. In truth both this brass work and the glasses themselves are generally made much too thin to be capable of being wrought true.

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in a vertical plane and direct the ſight to ſome object in the horizon under the ſun. Turn down the red glaſs and move the index till the reflected image of the ſun is in contact with the object ſeen directly. Fix now the index and turn the red glaſs round in its braſs cell: if this neither raiſes nor depreſſes the ſun's image, but it continues in contact with the object below as before, then the ſurfaces of the red glaſs are parallel. If turning this glaſs ſhould raiſe or depreſs the ſun's image, put the glaſs into that which leaves the image in the middle between the higheſt and loweſt points to which it ſhifts it, ſo that the refraction may remove the image of the ſun ſidewiſe along the horizon in a direction perpendicular to the plane of the octant. In this poſition of the red glaſs, its refraction will not make the inſtrument falſe*.

41. This examination is here directed to be made when the ſun is on the meridian, becauſe his diſtance from the object beneath does not vary ſenſibly at that time for ſeveral minutes; but it may be made at any other time, provided the plane of the octant be held in the plane of the hour circle paſſing through the ſun, and the object to be ſeen directly be placed in the interſection of that circle with the horizon.

42. The ſame trial of the red glaſſes, eſpecially the faint one, may be made by the moon, by bringing her reflected image into contact with ſome fixed ſtar, lying nearly in a great circle drawn through the moon, at right angles to her courſe in the heavens. For the diſtance from ſuch a ſtar will not be ſenſibly varied for many minutes by the proper motion of the moon in her orbit.

The METHOD of adjuſting the GLASSES and other PARTS of the OCTANT.

43. Before you begin to adjuſt the poſition of the glaſſes, you muſt firſt examine whether they are all faſt in their frames, if not you muſt fix them by the ſcrews behind. See par. 6 and 9. But take care not to force theſe ſcrews ſo as

* If the two ſurfaces of the red glaſs be not exactly parallel, the rays of light (in paſſing between the index and horizon glaſſes) will not be parallel before and after refraction at the red glaſs. But if the interſection of the ſurfaces of that glaſs, when produced, be parallel to the plane of the octant, this want of parallelism in the rays will not occaſion any error in the angle obſerved; for the refraction of the glaſs will then be wholly perpendicular to the plane of the inſtrument.

to bend or break the glaſſes. See likewiſe that the braſs work in general is every where firm and faſt, that the ſeveral adjuſting ſcrews which act againſt each other are juſt tight and no more.

To adjuſt the Index Glaſſes.

44. We ſhall begin with the index glaſs. Now to adjuſt the index glaſs is to ſet its plane pependicular to the plane of the inſtrument. This is performed by means of two adjuſting tools made of braſs and repreſented in perſpective, Fig 10 and 11. One of theſe tools, Fig. 10, conſiſts of two planes joined at right angles to each other, one of which we ſhall call the horizontal plane, the other the vertical plane, from their poſitions when in uſe. In the vertical plane is a ſmall hole for the eye to look through. The other two, Fig. 11 conſiſts of an horizontal plane like the former, at each end of which, and cloſe to one ſide, is an upright pillar: between the two pillars is ſtretched horizontally a ſilver wire or white horſe-hair exactly at the ſame height with the ſight-hole of the other tool; ſo that when the tools are ſet cloſe together and their vertical planes touch, the wire in one beſects the hole in the other.

45. To adjuſt the index glaſs then, firſt take the fore-ſight vane clear off the octant, ſo that the index may paſs over the place of that vane. Set the former of theſe tools, Fig. 10, on the arch or limb, ſet the latter, Fig. 11, on the ſides of the octant as near the index as may be, but not upon the index itſelf. Turn now the index clear off the limb, ſo that by looking through the hole in the former tool, Fig, 10, you may ſee that hole by reflection from the index glaſs; at the ſame time you are to ſee the wire of the other tool, Fig. 11, directly. If the reflected image of the hole you look through is biſected by the wire, then the index glaſs is upright. If the hole appears above the wire, then the index glaſs inclines too forward; contrariwiſe, if the hole appears appears below the wire. In this caſe the adjuſting ſcrews (par 7 and 8) muſt be the one releaſed the other tightened, till the hole ſeen by reflection is biſected by the wire.

46 It may be proper to repeat this examination, ſetting the tool, Fig. 10, that has the eye-hole, on a different part of the

the limb. If the plane of the inſtrument is perfectly flat, the reflected hole will appear biſected by the wire wherevever that tool is placed: but great preciſion in this particular muſt not be expected, and indeed need not be required.

47. In this adjuſtment take care always to releaſe one ſcrew before you ſet up the other that draws (or otherwiſe puſhes) againſt it. Likewiſe be ſure to leave all the adjuſting crews tight, by ſcrewing them up if ſlack, ſo that at laſt they may draw with a moderate force againſt each other*.

To adjuſt the fore Horizon Glaſs.

48. Two adjuſtments belong to the fore horizon glaſs, one to ſet it perpendicular to the plane of the octant, the other to make it agree with the diviſions on the limb. The former of theſe is done thus.

The index-glaſs being firſt adjuſted as before, bring o on the Vernier exactly againſt o on the limb, that is ſet the index to the beginning of the diviſions, † Hold now the plane of the octant parallel to the horizon, and obſerve the image of any diſtant object in the horizon at land, or at ſea the horizon itſelf. If the image ſeen by reflection is higher than the object itſelf ſeen directly, releaſe the fore ſcrew mentioned par. 11, and ſet up the back ſcrew, and contrariwiſe if the image ſeen by reflection is loweſt, till both are of an equal height, ſo that by ſhifting the index you can make both images exactly agree and appear as one object. In making this adjuſtment obſerve the caution before given par. 47.

49. If the ſea horizon be uſed for this adjuſtment, hold the plane of the octant parallel to the horizon as before, and obſerve whether the two images of the horizon coincide. This will be known by remarking whether the edge of the ſea, ſeen in the ſilvered part of the glaſs, joins in one ſtraight line with the edge of the ſea, ſeen through the tranſparent part of the glaſs. If the former appears either higher or lower than the

* Theſe adjuſting ſcrews ought to move very freely, otherwiſe it is impoſſible to feel how much they draw againſt each other. Their threads ough to be deep that the ſcrews may not be forced out, and fine that they may not releaſe themſelves; —but they are commonly very ill made.

† Some octants have a ſmall braſs button on the edge of the ſide. If this be turned up, it will ſtop the index juſt at the beginning of the diviſions.

latter, then the horizon glaſs muſt be adjuſted by the rule in the laſt paragraph.

50. This adjuſtment may be made by means of the ſun, moon, or a ſtar, in which caſe the octant may be held perpendicular to the horizon. If the image ſeen by reflection in that caſe appears on one ſide, to the right or left of the object ſeen directly, then the horizon-glaſs wants to be adjuſted. In whatever plane the octant be held, if the reflected image of the ſtar appears above the direct image *relatively to the plane of the octant*, then releaſe the fore ſcrew and tighten the back ſcrew, and contrariwiſe.

If this adjuſtment be made by a ſtar, ſlide the index backwards and forwards; obſerve if the reflected image in its motion paſſes over, that is upon the direct image. If not, the horizon-glaſs muſt be adjuſted as before till it does, ſo that, by ſtopping the index in the proper place, both images may preciſely coincide, and the ſtar appear as one.

51. The other adjuſtment of the horizon-glaſs is the ſetting it ſo that the index may ſhow upon the limb the true angle between the objects. This is by far the moſt important of all the adjuſtments, and is often ſtyled emphatically *adjuſting the inſtrument*. This adjuſtment cannot always be done immediately before an obſervation, and if done ſome time before is liable to alter, ſo that the moſt experienced obſervers adviſe us not to try at an exact adjuſtment, but to find the error occaſioned by the want of ſuch an adjuſtment, and allow for it in the calculation. We ſhall lay down both the method of adjuſting the octant perfectly, and of finding the error when this adjuſtment is not perfect, premiſing that all the other adjuſtments of the glaſſes before mentioned are to be made firſt, and as perfect as poſſible.

52. To *adjuſt* then the fore horizon-glaſs, turn the index till 0 on the Vernier ſtands exactly againſt 0 on the limb, and fix the index there.* Direct now the ſight to the moon, a ſtar, ſome diſtant object, and obſerve whether the image ſeen by reflection coincides perfectly with the object itſelf ſeen directly, ſo as to form one appearance; if not, bring them into one by

* Some octants have a ſmall braſs button on the edge of the ſide. Turn this up and the index will ſtop againſt it juſt in the proper place for this adjuſtment.

horizon-

turning the adjuſting lever belonging to the horizon-glaſs par. 12, and then fix the lever by the button-ſcrew.

53. If the ſea horizon be uſed for this adjuſtment, then hold the octant in a vertical poſition, and obſerve whether the edge of the ſea, ſeen in the ſilvered part of the glaſs, joins in one ſtraight line with the edge of the ſea ſeen through the tranſparent part; if not, bring them into one ſtraight line by turning the adjuſting lever, then fix that lever as before and the octant is adjuſted.

To find the Error of this Adjuſtment, commonly called the INDEX ERROR.

54. *Method* 1. Turn the index about till the moon, a ſtar, or ſome diſtant object, as before, appear as one by the union of the reflected image with the object itſelf ſeen directly, then the number of minutes by which 0 on the Vernier differs from 0 on the limb is the error of the adjuſtment, or the *index error*. If 0 on the Vernier ſtands between 0 and 90 on the limb, that is on the quadrantal arch (ſee par. 20) then this error is to be ſubtracted from the angle ſhown on the limb by the index; but if 0 on the Vernier ſtands on the other ſide of 0 on the limb, that is on the arch of exceſs, then the index error is to be added to the angle ſhown on the limb.

55. To find the index error by the ſea horizon, hold the octant in a vertical poſition as before, and turn the index till the edge of the ſea, ſeen in the ſilvered part of the horizon glaſs, joins exactly in one ſtraight line with the edge of the ſea ſeen through the tranſparent part, and the number of minutes by which 0 on the Vernier differs from 0 on the limb is the index error to be added or ſubtracted according to the rule juſt before laid down, par 54.

In reading off the minutes ſhown on the arch of exceſs, remember the caution in par. 23.

56. *Method* 2. Take the diameter of the ſun both on the quadrantal arch and arch of exceſs, half their difference is the index error. * This error is to be ſubtracted, if the diameter meaſured on the quadrantal arch exceeds the diameter meaſured on the arch of exceſs; if contrariwiſe, this error is to

* Half their ſum is the ſun's true diameter.

be added to the angle ſhown on the limb. In taking the ſun's diameter on the arch of exceſs remember the rule in par. 23, relative to reading off the odd minutes by the Vernier.

It will be proper on this occaſion to uſe the teleſcope. Whether that or plain ſights are uſed, it will be neceſſary to apply a ſmoked glaſs to defend the eye from the rays of the ſun.

The index error may be found in like manner by taking the diſtance of two ſtars, or taking the diameter of any diſtant land object, both on the quadrantal arch and arch of exceſs.

57. To prove any of theſe methods, ſet the index ſo as to ſhow the exact error found by that method; and to ſhow it upon the quadrantal arch if that error is to be ſubtracted, but to ſhow it upon the arch of exceſs if that error is to be added. Then fix the index, and directing the ſight to ſome diſtant object, take notice whether the direct and reflected images perfectly coincide or not. If they do, the index error is truly determined; if not, it muſt be corrected by the methods before laid down.

58. The horizon glaſs may likewiſe be adjuſted within doors by means of *two* near objects; but as in the diſtant ones a ſingle object is ſufficient, it will never be difficult to find one proper for this purpoſe abroad.

To adjuſt the back horizon glaſs.

59. The horizon glaſs for the back obſervation requires the ſame two adjuſtments as the fore horizon glaſs. The former of theſe, namely the ſetting it perpendicular to the plane of the inſtrument, is to be done in a manner like that directed for the fore horizon glaſs, par. 48. Firſt take the fore-ſight vane clear off the octant, ſo that that the index may paſs over the place of the vane. Hold the plane of the octant parallel to the horizon, and looking through the back vane (and through the tranſparent ſlit in the horizon glaſs) direct the ſight to ſome diſtant object in the horizon. Turn now the index clear off the limb till you ſee the image of the ſame object by reflection from the ſilvered part of the horizon glaſs, and make this image coincide as near as poſſible with the object itſelf ſeen directly. If by moving the index you can make both image and object coincide perfectly, then the

glaſs

glaſs needs no adjuſtment of this ſort; but if the reflected image ſtands a little above or a little below the object ſeen directly, then the glaſs wants to be adjuſted. Releaſe then one of the adjuſting ſcrews (par. 11 and 14) and ſet up the other till both image and object are of an equal height, ſo that by ſhifting the index you can make them both agree and appear as one ſingle object. Obſerve the caution before given, par 47.

This adjuſtment may likewiſe be made by the ſea horizon, or heavenly bodies, in a manner ſo perfectly reſembling that deſcribed in par. 49 and 50, that we need not repeat it.

60. The other adjuſtment of the back horizon glaſs is the ſetting it ſo that the index may ſhow upon the limb the true angle between the objects. This is equally important with the correſponding adjuſtment of the other horizon glaſs. What was obſerved of that in par. 51 may be applied to this. As this adjuſtment of the back horizon glaſs is very important, and yet but little known, we ſhall be particular in deſcribing ſeveral methods of doing it both at land and ſea; and ſhall likewiſe ſhow how to find the index error when this adjuſtment is not perfect.

Methods to be practiſed at land.

61. *Method firſt.* Look out for a part of the horizon that has two diſtant objects upon it, on two points of the compaſs either exactly oppoſite or nearly ſo. Turn the index till o on the Vernier ſtands exactly againſt o on the diviſions on the limb, and fix the index there. Hold now the plane of the octant level with its face upwards, and, directing the ſight to one of theſe objects, look in the ſilvered part of the horizon glaſs for the reflected image of the other which is then behind you. If the direct and reflected objects do not exactly coincide, change your place, moving nearly at right angles to the line joining the two objects, till they do coincide. Invert now the octant, that is, turn its face downwards, but hold its plane level. Direct the ſight to the ſame object as before, and obſerve if the object behind you (ſeen by reflection) exactly coincides with that ſeen directly which is before you. If the two objects coincide, now the octant is inverted, as they did before, the horizon glaſs needs no adjuſtment. If

the

the objects do not coincide, turn the adjusting the lever till you bring the object seen by reflection half way towards that seen directly. Shift now your place a little, as before, till the objects again perfectly coincide, the face of the octant being still downwards. When by shifting your place the objects are made to coincide in that position of the octant, then turn its face upwards as at first. If the objects now coincide also, the adjustment is perfect; if not, by means of the adjusting lever, bring the object seen by reflection half way towards that seen directly, and repeat the whole process as before.

62. By *distant objects*, in this particular process, are to be understood objects not more than four or five miles distant from the observer; otherwise shifting his place, as before directed, will not make them coincide. Nor should these objects be nearer than half a mile: for then any little difference in the way of holding the octant will alter the coincidence, even though the observer stands on the same spot. *

63. In all this process the observer is constantly to keep his face towards one and the same object, which is always to be seen directly, and the other object always by reflection.

64. It will be convenient always to observe with the left eye when the face of the octant is upwards, and with the right eye when the face of the octant is downwards. The contrary may be done, but there is danger lest the hair or wig of the observer should hide the object to be seen by reflection, which lies behind him.

To find the error of the adjustment or index error.

65. *Method first.* Having found two distant objects in the horizon, † nearly on opposite points of the compass, hold the octant level with its face upwards and direct the sight to one of these objects, then turn the index till the other object (which is behind you, and must therefore be seen by reflection) coincides with that seen directly, and mark the degrees and

* This adjustment might be better made by very distant objects without changing the place of the observer, provided the horizon was distinguished by such a variety of objects, that those parts of it which seem to coincide at each change in the position of the octant, could always be ascertained.

† The objects cannot be too far distant in this case.

minutes

minutes ſhewn on the limb. Turn now the face of the octant downwards, and directing the ſight to the ſame object as before, move the index till the other object ſeen by reflection coincides with that ſeen directly, and again mark the degrees and minutes ſhewn on the limb. Half the difference of theſe two arches, thus ſhewn on the limb, is the error of the adjuſtment. One of theſe arches will be a portion of the quadrantal arch, and the other a portion of the arch of exceſs. In reading off this laſt arch obſerve the rule in par. 23. In every back obſervation I ſuppoſe the angle between the objects to be eſtimated by the rule in par. 24; that is, the degrees and minutes contained in that angle are the ſupplement (to 180 degrees) of the degrees and minutes ſhewn on the limb, and eſtimated by the figures engraved thereon: But I ſuppoſe the ſmall arch aforeſaid to be read off in the uſual manner, by the figures on the limb. This premiſed, the error of the adjuſtment found as before is to be added to the angle between the objects, when the portion of the quadrantal arch exceeds the portion of the arch of exceſs. If contrariwiſe, this index error is to be ſubtracted.

66. To prove whether the index error has been rightly found, ſet the index to that number of degrees and minutes to which it amounts; but ſo as to ſhow it upon the quadrantal arch, if the larger of the two ſmall arches mentioned before, par. 65, was a portion of the quadrantal arch; otherwiſe ſet the index ſo as to ſhew the index error on the arch of exceſs, and fix the index there. Then, by the method in par. 61, obſerve whether the ſame two diſtant objects, on oppoſite points of the compaſs, coincide both in the direct and inverted poſitions of the octant; for they ought to do ſo, if the index error has been rightly determined.

67. *Method ſecond, for the adjuſtment.* Find two diſtant objects nearly on oppoſite points of the compaſs, as before directed in par. 61. Fix the index at 0, then direct the ſight to one of thoſe objects, and cauſe the reflected image of the other object to coincide with that ſeen directly, by changing your place, as was directed in par. 61. The objects being thus made to coincide, keep the face of the octant upwards, but turn yourſelf half round, ſo as to ſee that object directly which was before ſeen by reflection, and the other object by reflection which was before ſeen directly. If they now coincide

cide again, the horizon glaſs is truly adjuſted; if they do not coincide, turn the adjuſting lever to bring the object ſeen by reflection half way towards that ſeen directly. Proceed as in par. 61, only keep the octant conſtantly with its face upwards, and, inſtead of changing the poſition of the octant as there directed, change your own poſition by turning half round. In this method it is beſt to obſerve conſtantly with the left eye. The nearneſs of the objects is of leſs conſequence in this than in the laſt method; it will be of none, if the directions for making this adjuſtment within doors be followed abroad. See par. 72.

68. *Method ſecond, for finding the index error.* Having found two diſtant objects * nearly on oppoſite points of the compaſs, direct the ſight to one of them, and turn the index till the other which is behind you coincides with that which is ſeen directly, and mark the degrees and minutes ſhewn on the limb. Keep the face of the octant ſtill upwards, but turn yourſelf half round as before mentioned, par. 67. Direct the ſight to the other object now before you, and turn the index till the two objects again coincide, and again mark the degrees and minutes ſhewn on the limb. Half the difference of theſe two arches is the error of the adjuſtment, and is to be added or ſubtracted according to the rules laid down in the latter end of par. 65.

69. This may be proved in par. 67, juſt as the firſt method of finding the index error was proved by the firſt method of adjuſting the horizon glaſs in par. 66.

70. In the firſt method the ſame object is conſtantly ſeen directly, and its oppoſite by reflection; which is a convenience when one object is bright and the other obſcure. In the ſecond method you avoid the trouble of holding the inſtrument with its face downward, and obſerving with it in an awkward poſition.

71. You may rectify the horizon glaſs (or may find the index error) either by changing the poſition as in the firſt method, or by changing your own poſition as in the ſecond method; but not if you change both the poſition of the octant and alſo your own. For whatever objects coincide before *both* theſe changes are made, the ſame will coincide

* Here the objects cannot be too far diſtant.

after,

after, and that whatever be the position of the horizon glass or index glass.

72. *Method third, for the adjustment; to be used within doors.* Upon an horizontal stand fix two conical points, whose distance is exactly that between the centers of the index glass and horizon glass; or, which is the same, the distance between the centers of the screws that fasten the index glass and the horizon glass to the plane of the octant. The heads of these screws appear on the back side of the octant; in the center of these you may countersink or turn conical holes to receive the points aforesaid. Provide two small objects that can be easily seen, and that are well defined. A line and plummet hung against two opposite windows by day, or two candles by night, may do. Set the stand nearly in the line joining these two objects. Set 0 on the Vernier to 0 on the limb of the octant, fix the index, and then place the octant upon the two points on the stand, making them enter the conical holes in the heads of the screws aforesaid. Turn the stand about in its own place, till you see one of the objects by direct vision through the unsilvered slit in the horizon glass; then look in the silvered part for the reflected image of the other object now behind you. If the direct and reflected objects do not coincide, make them do so, either by shifting the place of one of the objects, or by shifting the stand itself across the line joining the objects. When the direct and reflected images are thus made to coincide, then turn the octant half round, placing the screw of the index glass upon the point which before supported the horizon glass; and contrariwise. See if the objects now coincide; if they do, the horizon-glass is truly adjusted; if they do not, by means of the adjusting lever bring the object seen by reflection half way towards that seen directly. Then move either one of the objects, or the stand itself, so as to make the two objects coincide perfectly. Turn the octant again half round, placing it on the points as at first. See if the objects, viewed as before, again coincide; if not, repeat this operation till they do, and the horizon glass will be perfectly adjusted.

73. In making this adjustment care must be taken, *first*, that the objects lie in the plane of the octant, and, if lines with a plummet are used, the plane of the octant should be

placed nearly level, otherwiſe the lines, when ſeen by the octant, cannot be made to coincide perfectly, but will always ſeem to croſs each other. But *ſecondly*, when the objects are within a few feet of the octant, particular care muſt be taken that the reflections be always made from the middle of the glaſſes, eſpecially the reflection from the index glaſs. To be ſure of this, cut a card to fit the inſide of the frame that holds the index glaſs ſo as to bed flat againſt it. Acroſs the middle of this card, draw a black line, ſo that, when the card is applied cloſe to the index glaſs, this black line may be perpendicular to the plane of the octant. This line, when the card is ſo applied, may be ſeen by reflection from the horizon glaſs. It will be eaſy, by twiſting the ſtand about and ſhifting the place of the eye, to make this black line and the object ſeen directly to coincide, and that in the very middle of the horizon glaſs. Take away the card, and then, if the image of the other object be made to coincide with that ſeen directly, it muſt be by reflection from the middle of each glaſs: otherwiſe a thread may be ſtretched acroſs the middle of the index glaſs, perpendicular to the plane of the octant, and another thread in like manner acroſs the middle of the horizon glaſs. By twiſting the ſtand round, and placing the eye properly, both object may be made to coincide with each other, and with the threads in the middle of each glaſs; and then it will be of no conſequence how near the objects are to the octant. In this method it is beſt to obſerve with the left eye.

74. If in the ſecond method by diſtant objects, par. 67, the octant was to be placed on a ſtand in the manner here deſcribed, that adjuſtment would be more perfect, and the beſt of all the methods here laid down.

75. *Method third, for finding the index error, to be uſed within doors.* Proceed juſt as before in par. 72, only inſtead of turning the adjuſting lever to make the object ſeen by reflection move half way towards that ſeen directly, turn the index itſelf. When the two objects are at laſt made to coincide in both poſitions of the octant, the index will then ſhow on the limb the index error, which is to be added if this error be ſhewn on the quadrantal arch, and ſubtracted if it be ſhown on the arch of exceſs. See the latter end of par 65.

76. *Method fourth, for the adjuſtment.* Look out for two objects that ſubtend an angle of 90 degrees, or near it; adjuſt the horizon glaſs for the fore obſervation by par. 52, and, looking through the foreſight vane, at one of thoſe objects directly, turn the index till the reflected image of the other object coincides with it, and mark the degrees and minutes ſhown on the limb. Set the index to ſhow that number of degrees and minutes by the back obſervation: That is, ſet the index ſo as to ſhow, by the figures on the limb the ſupplement of the number of degrees and minutes before ſhown in the fore obſervation, (ſee par 24). Then looking through the back-ſight vane, ſee if the ſame two objects coincide in the back horizon glaſs; if not, turn the adjuſting lever of this glaſs till they do coincide, and then this glaſs alſo will be adjuſted.

77. *Method fourth, for finding the index error.* Having found two objects that ſubtend an angle of 90 degrees, or a few degrees over or under as before, take the exact angle between theſe objects by the fore obſervation, making due allowance for the index error according to par. 54 or 56. Again, take the angle between the ſame objects by the back obſervation, and the difference of theſe angles ſo taken is the index error in all back obſervations, and is to be added or ſubtracted as the angle taken by the fore obſervation (when properly corrected) is greater or leſs than the angle between the ſame objects taken by the back obſervation.

78 The laſt method ſuppoſes the whole octant (or the extent of 90 degrees on the limb) to be an exact eighth part of a circle. The former methods of adjuſting the horizon glaſs depend not at all on the diviſions on the limb, and the methods of finding the index error only ſuppoſe the diviſions to be true for a very ſmall extent on each ſide of the point from which they begin. In the former methods the obſervations are made when the index is near 0, where the field of the inſtrument is the greateſt, and the objects eaſieſt found; in this laſt method the obſervations are made when the index is near 90 degrees, where the reflections are very oblique, and the field ſmall. It may be remarked too, that the object, which is ſeen directly in the fore obſervation, will be ſeen by reflection in the back obſervation. This is an inconve-

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nience, if both objects are not equally bright. By holding the octant with its face downwards in one of these observations, each object will be seen by the same means in both; but it is troublesome to hold the octant in that position.

The method of adjusting the Back Horizon Glass, and of finding the error of that adjustment at SEA.

79. *To adjust this glass.* Set the index so as to show, on the arch of excess, double the angle of the depression or dip of the sea horizon.* Then holding the octant in a vertical plane, turn the adjusting lever till the edge of the sea, seen in the silvered part of the glass, coincides with the edge of the sea seen through the transparent slit of the horizon glass. Fix the adjusting lever in that place, and the octant is adjusted.

80. In this position of the octant, the object seen by reflection, which is behind the observer, will appear to him to be before and inverted. Thus in the present instance the water will appear above and the sky below. If the two edges of the sea (that seen directly and that seen by reflection) cannot be made to touch all along, but cross each other, it is a sign that the octant is not held in a vertical plane. Its position must be altered, till the two edges of the sea so seen are parallel, and then the adjusting lever will bring them to touch all along.

81. If the index glass and horizon glass are too near each other, the head of the observer will be in the way. The best construction of this octant will not allow the observer to have his hat on, or any thing high upon his head.

82. *To find the index error.* Hold the octant in a vertical plane, the index glass being uppermost as usual. Turn the index till the two edges of the sea (one seen directly and the other by reflection) meet each other and touch all along; then mark the degrees and minutes shown by the index on the limb. Invert now the octant, so that the arch may be uppermost, but the index glass and centre of the instrument lowest; hold-

* See a table of the depression or dip of the horizon of the sea, among the tables belonging to the Nautical Ephemeris, published by order of the Commissioners of Longitude.

in

ing the arch with your left hand, and ſupporting the lower part of the octant with your right, yet ſo as not to touch or ſhade any of the glaſſes with your hand. Let the face of the octant be turned towards you, but keep the plane of the octant vertical. Incline now your head ſidewiſe till you bring your right eye down to the ſight vane, and, looking through it, move the index by the left hand, till the two edges of the ſea (that ſeen directly and that by reflection) agree as before, and again mark the degrees and minutes ſhown on the limb; half the difference of theſe two arches is the index error. * The index error ſo found is to be added or ſubtracted according to the directions in par. 65.

83. That rule given in par. 65 holds good in taking diſtances between the heavenly bodies, or in taking angles between land objects; but in one ſort of obſervation, namely, *taking the ſun's altitude by the back obſervation*, the index error, found as before, follows a contrary rule. It is to be ſubtracted from the ſun's altitude, when the portion of the quadrantal arch exceeds the portion of the arch of exceſs: if contrariwiſe, this index error is to be added. The reaſon of this is becauſe the figures engraved on the limb immediately ſhow the ſun's altitude in the back obſervation, juſt as they do in the fore obſervation. See par. 25.

84. It may be of ſome uſe to ſuſpend the octant by a line tied round the arch. This line bearing the weight of the octant, it may be more eaſily guided into the proper poſition by the right hand, while the index is ſet by the left. If the obſerver chuſes to make uſe of his left eye rather than the right, then the back of the octant muſt be held towards him, and whatever was before directed to be done with the right hand muſt now be done with the left, and contrariwiſe.

It muſt be confeſſed that this method of finding the index error is rather troubleſome. Poſſibly that before deſcribed, par. 75, may be put in practice on ſhipboard. If no other way can be practiſed, the mariner may take the ſun's meridian altitude by the back obſervation immediately after he has compleated his fore obſervation for the latitude. When all the neceſſary corrections are made, except that for the

* One fourth of the ſum of theſe two arches is the dip of the ſea horizon.

index

index error ſought, then compare the ſun's true altitude deduced from the fore obſervation, with the like reſult from the back obſervation: Their difference may be conſidered as ariſing wholly from the wrong poſition of the back horizon glaſs, and may be applied accordingly to correct this latter obſervation of the ſun's altitude. This correction muſt be applied contrariwiſe in taking diſtances between the heavenly bodies, as was before remarked, par. 83.

To adjuſt the place of the ſight vanes, and other parts of the octant.

85. We ſeldom find the ſight vanes ſet exactly in their true place. Their true place is that which gives the largeſt field. This depends on the placing of the index glaſs upon the index, and the point of the limb from whence the diviſions take their beginning. We ſhall ſhow *firſt* the method of finding the true place of each ſight vane, ſuppoſing the index glaſs to be already fixed on the index, and the point from whence the diviſions are to commence to be already marked upon the limb. *Secondly*, To find the point of the limb from whence the diviſions ought to take their beginning, ſuppoſing the place of the vane determined, and the index glaſs fixed on the index. *Thirdly*, How to fix the index glaſs properly upon the index, ſuppoſing the ſight vane already placed, and the point determined from whence the diviſions are to begin.

86. For the firſt, adjuſt the index glaſs by par. 44. Set the index to the beginning of the diviſions. Find the place of the ſight vane nearly, which you will eaſily do by moving the eye about till you can ſee ſome object by the double reflection. Adjuſt now the horizon glaſs exactly by par. 49, 52; then ſhift the ſight vane till the center of the index glaſs is ſeen by reflection from the middle of the horizon glaſs, and that will be the true place of the ſight vane.

87. You may uſe the adjuſting tool, fig. 10, ſee par. 44, by way of a moveable ſight vane, and you may ſtretch threads acroſs the index and horizon glaſſes to diſtinguiſh their middle, as was propoſed par. 73. But the beſt way is as follows. Firſt turn the index to 90 degrees, ſo that the whole frame of the index glaſs may be ſeen as an object by reflection from

the

the horizon glaſs: then turn the index gradually back again, and at the ſame time ſhift the place of the ſight vane till the reflected image of the frame of the index glaſs juſt fills up the whole area of the horizon glaſs; the two ends of the frame of each glaſs apparently coinciding; and the ſight vane will then ſtand in its true place.

The place of the ſight vane for the back obſervation ought to be adjuſted very exactly, becauſe the field is generally very ſmall.

88. For the ſecond point propoſed, Turn the fore horizon glaſs by the adjuſting lever till the center of the index glaſs is ſeen through the fore-ſight vane, by reflection from the middle of the horizon glaſs, or till the reflected image of the frame of the index glaſs can be made juſt to fill up the area of the horizon glaſs as before. Turn now the index till the reflected image of ſome diſtant object coincides with that object itſelf as deſcribed in par. 52, and that will be the place of the index at the beginning of the diviſions. This found, the point from whence the diviſions ought to commence may be eaſily marked on the limb. This point might have been determined from the aſſumed place of the back-ſight vane, but it is beſt to determine it from the place of the fore-ſight vane, and then to find the place of the back-ſight vane from this point ſo determined.

89. The common octants are, I believe, made by one workman, and then ſent to another to be graduated; perhaps without marking out where the diviſions ſhould begin. If he who graduates the arch ſhould happen to begin at ſome diſtance from the proper point, (which depends upon what the former workman has done) half the field may be loſt when the index ſtands at 90 degrees.

90. For the laſt point propoſed, ſet a fine pin upright on the index and in the center on which it turns. Move the fore horizon glaſs by the adjuſting lever, till this pin is ſeen by reflection from the middle of the horizon glaſs. Set the index to the beginning of the diviſions; then placing the index glaſs upon the index with its outer ſurface cloſe to the pin, turn the glaſs about till the reflected image of ſome diſtant object coincides with that object ſeen directly. Draw now a line upon the index along the outer ſurface of the glaſs. Draw alſo another line on the index parallel to the former

former, between that and the index glaſs; and diſtant from the firſt line by two-thirds of the thickneſs of the index glaſs; and this laſt line will give the true place of the outer ſurface of the index glaſs.

Otherwiſe. Having drawn the firſt of thoſe lines before mentioned, deſcribe a circle upon the index whoſe center is the point on which the index turns, that line being a diameter. Draw alſo ſeveral radii making equal angles *reſpectively* with that diameter. Then if the index glaſs be placed upon that diameter, and be moved a little backwards and forwards parallel to it, you will ſee in one ſituation each of theſe radii united with the reflected image of its correſponding radius, ſo as to make one continued line forming ſo many diameters interſecting in one common point. Draw now a line upon the index along the outer ſurface of the index glaſs when in this poſition, and that will be its true place.

Having found the line on which the outer ſurface of the index glaſs is to be placed; ſlide the glaſs along that line till its middle is ſeen by reflection from the middle of the horizon glaſs, or till the frame of the index glaſs (conſidered as an object) ſeen by reflection from the horizon glaſs, can be made to fill the area of the horizon glaſs in the manner before deſcribed, par. 87.

Rules for conſtructing an Octant.

91. The foregoing rules ſuppoſe ſome circumſtances in the octant already determined on which thoſe rules are founded. No very preciſe rules can be given for ſetting out an octant intirely from the beginning, but the general principles upon which it ought to be conſtructed may be laid down. Now one principle is, that very oblique reflections from the mirrors ought to be avoided. For this reaſon the center of the horizon glaſs ſhould be placed, as near as poſſible, in a line drawn through the center of the index glaſs, and perpendicular to its ſurface when the index ſtands at the beginning of the diviſions. I ſay *as near as poſſible*; for the horizon glaſs cannot be placed in this very line. In ſuch a caſe it would ſtand between the index glaſs and the object which ſhould be ſeen by reflection, and ſo ſkreen this object from the ſight.

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The horizon glaſs itſelf ought not to be ſeen by the double reflection, and yet ſhould be but juſt out of ſight. Accordingly, when the index is turned a little way off the limb, the fore horizon glaſs ſhould be ſeen by its own reflection, but not while the index is on the limb.

92. The place of the fore horizon glaſs being once determined, its poſition, and the poſition of its axis of viſion, follows of courſe. The former muſt be parallel to the index glaſs when the index ſtands at the beginning of the diviſions: the latter, namely the axis of viſion, will be found by ſuppoſing the line, joining the centers of the two glaſſes aforeſaid, to be a ray incident on the ho izon glaſs; for the reflected ray is the axis of viſion, agreeably to par. 18 and 86.

93. To place the back horizon glaſs properly is a critical buſineſs. Care muſt be taken that the fore horizon glaſs does not intercept the rays paſſing between the index glaſs and back horizon glaſs; for which reaſon this glaſs cannot ſtand in the line drawn through the centers of the other two glaſſes, but on one ſide that line. And it muſt ſtand the ſame way from that line, that o on the limb ſtands from 90; otherwiſe this (back) horizon glaſs would intercept the view of ſome of the objects which ſhould be ſeen by reflection in the fore horizon glaſs. The nearer the back horizon glaſs ſtands to that line, the more oblique will be the reflections at this glaſs, and the leſs oblique the reflections at the index glaſs. The latter is of moſt conſequence, becauſe of the great length of the index glaſs, neceſſary to take in a moderate field, when theſe reflections are very oblique: therefore though the back horizon glaſs ſhould ſtand on one ſide the line before mentioned, to clear the fore horizon glaſs, yet it ſhould be but juſt clear of it. The back horizon glaſs muſt be ſo far diſtant from the index glaſs that the head of the obſerver may not be in his own way; and no more: for the greater this diſtance the leſs will be the field. The back horizon glaſs always ſtands at right angles to the fore horizon glaſs, whence the poſition of its axis of viſion may be determined as before, par. 92, at the end.†

* It may be convenient ſo to place the index glaſs upon the index, that its plane continued may cut the index down the middle. The weight of the index, when turned off the limb to adjuſt the index glaſs, (par. 44) will not then alter the inclination of that glaſs to the plane of the octant.

† It may not be amiſs to ſet down the following mathematical rules, omitting their demonſtration.

94. The foregoing rules being conſidered, the plan of a good octant will be found to lie within narrow limits. The deſign firſt publiſhed in the Tranſactions, Nr. 420, and generally followed ever ſince, will ſcarce admit of any alterations. Hadley ſeems to have underſtood the theory ſo perfectly, and to have conſulted practical convenience ſo fully, that notwithſtanding the pretenſions of ſeveral, no real improvements have been made in the way of placing the glaſſes or in the general conſtruction of the inſtrument.

95. It may be of ſome uſe to give here the proper diſtances between the ſeveral glaſſes, for any octant large enough to admit of the back obſervation, and likewiſe the ſeveral angles of incidence at each mirror. *

1. In the fore obſervation, the angle between the ray incident on the index glaſs and that ray after two reflections, is double the inclination of the faces of the ſpeculums; and this is the angle ſhown on the limb, when the glaſſes are perfectly adjuſted. This is likewiſe the angle which the (doubly) reflected image of any object makes with that object itſelf; and if that reflected image coincides with ſome other object ſeen directly through the tranſparent part of the horizon glaſs; then this is the angle which the object ſeen by reflection and the object ſeen through the glaſs would ſubtend to the naked eye, placed at the interſection of the incident ray with that ray after two reflections.—Theſe different expreſſions of the ſame quantity are here noted once for all.

2. In the back obſervation, the angle between the ray incident on the index glaſs and that ray after two reflections is double the ſupplement (to 180 degrees) of the angle which the faces of the ſpeculums make with each other. And this is the angle ſhown on the limb if it be read off by the rule in par. 24.

3. The angle which the line, joining the centers of the index glaſs and fore horizon glaſs, makes with a perpendicular to the index glaſs, when the index ſtands at the beginning of the diviſions, is equal to the angle of incidence on the index glaſs in that poſition; and this is the conſtant angle of incidence on the fore horizon glaſs

4. The angle of incidence on the index glaſs, when the index ſtands at 90 degrees, exceeds the former angle by 45 degrees.

5. In the back obſervation, the angle of incidence on the index glaſs is greater than the angle of incidence in the fore obſervation (the index ſtanding at the ſame place) by the angle which the centers of the two leſſer mirrors ſubtend at the center of the great one.

6. The angle of incidence on the back horizon glaſs is conſtantly the complement (to 90 degrees) of the ſum of the angle of incidence on the fore horizon glaſs, and the angle which the centers of the two leſſer mirrors ſubtend at the center of the great one.

7. The angle which the two axes of viſion (for the fore and back obſervation) make with each other, is equal to that which the two leſſer mirrors ſubtend at the center of the great one.

* The angles of incidence are reckoned from the perpendicular.

Inches

	Inches
Diſtance of the center of the index glaſs from the center of the fore horizon glaſs	4½
Center of the index glaſs from the center of the back horizon glaſs	6
The two horizon glaſſes from each other	2
Length of the index glaſs	2½
of the fore horizon glaſs	1
of the back horizon glaſs	1⅓

	D	M
The angle which the two horizon glaſſes ſubtend at the index glaſs	12	30
Angle of incidence on the index glaſs in the fore obſervation, the index ſtanding at the beginning of the diviſions. Alſo the conſtant angle of incidence on the fore horizon glaſs	15	30
Angle of incidence on the index glaſs in the fore obſervation, the index ſtanding at 90 degrees	60	30
The former of theſe angles in the back obſervation	28	00
The latter of theſe angles in the back obſervation	73	00
The conſtant angle of incidence on the back horizon glaſs	62	00

Of the Motions of the Reflected Image.

96. We ſhall here deſcribe the motions of the reflected image, while the octant is turned round ſome of the principal lines both perpendicular and parallel to its plane.

97. And firſt ſuppoſe the octant to be turned round the common interſection of the two reflecting planes, which will be a line perpendicular to the plane of the inſtrument, and the inſtrument itſelf will then move in its own plane round that axis. In this caſe the reflected image of any object will be perfectly at reſt. Let this image coincide with any object ſeen directly, and they will conſtantly ſtick together and be united notwithſtanding this motion of the octant; and that whether the object be near or diſtant, and whatever be the inclination of the reflecting planes to each other. This is the grand principle demonſtrated by *Hadley*, Philoſoph. Tranſ. Nr. 420. Almoſt all other writers have only tranſcribed his paper word for word. *

98. Secondly

* *Smith's* Optics. Obſervaciones Aſtronomicas, &c. por D. *Jorge Juan* y D. *Antonio de Ulloa*.

If two looking-glaſſes be joined together ſo as to form an acute angle, and an object be placed between them, a ſpectator will ſee a number of images of that object in each glaſs, all placed in the circumference of a circle whoſe center is in the common ſection of the two glaſſes, and whoſe radius is the diſtance of the object from that ſection. Theſe images are formed by an odd and an even number of reflections alternately. Thus let the glaſs into which you look be called *A*, and the other *B*; then the firſt image is formed by the reflection of *A* only. The next image is formed by two reflections, firſt from *B* and then from *A*. The third image is formed by three reflections; firſt from *A*, and then from

B,

98. Secondly. Suppoſe the octant turned round ſome other axis parallel to the former and therefore perpendicular to the plane of the inſtrument. In this caſe diſtant objects, ſeen by reflection, will have no motion at all; the reflected image of a near object will have a ſmall motion to parallel to the plane of the octant, but will keep its poſition parallel to itſelf.

99. Let now the axis of motion be parallel to the plane of the octant, and firſt ſuppoſe the axis of viſion to be the axis of motion. Then in the fore horizon glaſs if the reflected image of any object, whether near or diſtant, be made to coincide with that object ſeen directly, it will ever remain at reſt; the direct and reflected images of that object being perfectly united in every poſition of the octant. If the reflected image be ſeparated from the object by moving the index, the image will then deſcribe a circle whoſe center is that object itſelf, and it will move the ſame way and with the ſame angular velocity that the octant does; but the reflected image will keep its poſition parallel to itſelf and to the original object.

100. Hence, in the vibratory motion of the octant round the axis of viſion, the reflected image will deſcribe an arch of a circle (to and fro) convex towards thoſe objects that lie at a greater angular diſtance from the original object than its reflected image does, that is ſuch as are further diſtant than the angle ſhown on the octant.* In this vibratory motion the image keeps its poſition parallel to itſelf, and it croſſes the plane, paſſing through the axis of motion and the object, at right angles.

B, and then from *A* again, and ſo on. If now the two glaſſes, keeping their inclination, be turned round their common ſection as an axis; theſe images will alternately move and be at reſt. All the images formed by an odd number of reflections will move round their common center with double the angular motion of the glaſſes, whilſt the images formed by an even number of reflections will remain perfectly fixed

This experiment is not only entertaining, but ſuggeſts an eaſy proof of the property of the octant here mentioned. For when the number of reflections is ever, the diſtance of any of thoſe images from the original object, meaſured along the circumference of the circle in which they all ſtand, is equal to the arch (of that circle) intercepted by the reflecting planes, multiplied by the number of reflections. Therefore as the quantity of the intercepted arch is not changed by the common motion of the planes round their angular meeting, the diſtance of none of theſe images from the fixed object will be altered, and conſequently they will all remain at reſt; and among them, that formed by two reflections only,

* See the note on par. 93, Nr. 1.

101. In

101. In the back horizon glaſs, when the index ſhows 180 degrees and the glaſſes make a right angle, if the octant be turned round the axis of viſion, the reflected images of the ſeveral objects behind you will deſcribe concentric circles round the axis of viſion, and the common center of theſe circles will be that object before you, to which the axis of viſion is directed, or that point of the reflected image which coincides with this object. The whole reflected image therefore changes its poſition juſt as the ſails of a windmill do. The motion of this image is the ſame way with that of the octant, but has double its angular velocity. When the plane of the octant is horizontal, this reflected image is upright; when the octant has been turned round 90 degrees, ſo as to come into a vertical plane, the reflected image will then be turned round 180 degrees, that is it will be inverted.

102. If the index be removed from 180 degrees (ſo as to increaſe the angle between the ſpeculums) and the octant be made to vibrate about the axis of viſion, the reflected image will deſcribe the arch of a circle convex towards ſuch objects as are further diſtant from the original object than the angle ſhown on the octant, reckoning that diſtance the ſame way that the image is removed from the object. The image in this caſe will continually change its poſition, and it will croſs at right angles a plane paſſing through the axis of motion and the original object.

103. Let the octant be made to vibrate about an axis a little inclined to this axis of viſion, but in the ſame plane as before; for inſtance, let it be made to vibrate about the axis of viſion belonging to the fore horizon glaſs. Let the index ſtand at 180 degrees as before, the reflected image, ſeen in the back horizon glaſs, will deſcribe an arch of a circle whoſe center will be that object to which the axis of motion is directed. If the index ſtands at any other place, the reflected image will deſcribe an arch of a circle convex the ſame way as deſcribed before, par. 102. In both theſe caſes the image changes its poſition as well as its place. In theſe vibratory motions, the angular velocity of the image depends on the place of the index. At 90 degrees it is nearly equal to that of the octant, at 180 degrees nearly double.

104. Let now the axis of motion be perpendicular to the axis of viſion, but parallel to the plane of the octant, and let the

the index ſtand at 0 in the fore obſervation, that is at 180 in the back obſervation. Then in the fore horizon glaſs the reflected image will have no motion. In the back horizon glaſs the reflected image will turn round the axis of motion with double the angular velocity of the octant. In this laſt caſe the image will keep its poſition parallel to itſelf. Thus if the octant be held in a vertical plane, and the image of a ſteeple behind you be ſeen in the back horizon glaſs; then, if the octant be turned about the forementioned axis, that image will be carried round the horizon. The ſteeple will indeed appear inverted, according to par. 101, but will keep that poſition, and move parallel to itſelf round the horizon.

105. If the index ſtand at 90 degrees, and the octant be turned about the forementioned axis, the reflected image will be carried round the axis with the angular velocity of the octant, and the poſition of the image will likewiſe be changed with the ſame angular velocity. Thus let the plane of the octant be held horizontal, and the index ſet at 90 degrees; let the octant be turned round an axis parallel to the horizon, but perpendicular to the axis of viſion. Then the reflected image of a ſteeple, whether ſeen in the fore or back horizon glaſs, will be carried round the axis of motion, ſo that when the octant has deſcribed 90 degrees, and is come into a vertical plane, the ſteeple will appear either in the Zenith or Nadir (according to the direction in which the octant is turned) but inſtead of being erect the ſteeple will appear parallel to the horizon. In intermediate places of the index the image will partake more or leſs of thoſe motions, which it has when the index is at either extremity of the arch.

106. The ſeveral axes round which the octant is ſuppoſed to vibrate in the laſt ſeven paragraphs are all ſuppoſed to be parallel to the plane of the octant, but to paſs through the axis of viſion, interſecting it at different angles. If, inſtead of this, we ſuppoſe the ſeveral axes of motion to lie in the plane of the octant itſelf, and to be reſpectively parallel to the ſeveral axes of motion in the former caſe, then the apparent motion of the reflected image in each caſe will be the ſame as before, provided the original object be not too near the octant.

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The APPLICATION of the OCTANT to ſeveral PURPOSES.

General Rules in making Obſervations.

Directions how to hold the Inſtrument.

107. The beſt way of holding the octant is to ſupport its weight chiefly by the right hand, and to reſerve the left to govern the index. Place the thumb of the right hand againſt the edge of the octant, under the ſwelling part of which the fore-ſight vane ſtands, and extend your fingers acroſs the back of the octant ſo as to lay hold on the oppoſite edge, placing the fore finger above and the other fingers below the ſwelling part on which the fore horizon glaſs ſtands. Thus you may ſupport the inſtrument conveniently by the right hand only, eſpecially when in a vertical poſition. By reſting the thumb of the left hand againſt the ſide, or the fingers againſt the middle bar, you may draw the index either to or from you gradually; but if the index is furniſhed with an adjuſting ſcrew you will be able to ſet it more exactly; ſtill more ſo if the octant be ſupported by a ſtand. In taking the diſtance of the moon from the ſun or ſtars to find the longitude at ſea, the weight of the inſtrument is frequently ſupported by a ſtaff, the bottom of which reſts on a belt, or in the waiſtcoat pocket of the obſerver. At the top of this ſtaff is a ball and ſocket, upon which the inſtrument is fixed, and by means whereof it can be readily put into the proper plane for obſerving.

The foregoing directions relate to the fore obſervations. You muſt hold the inſtrument in like manner for the back obſervation; only here you ſhould ſupport the inſtrument by the left hand, and govern the index by the right.

Of the proper plane, in which all obſervations are to be made.

108. In all obſervations care muſt be taken to hold the octant in the proper plane, namely the plane which paſſes through the eye of the obſerver and the two objects; that is,

in

in aſtronomical obſervations, in the plane of a great circle paſſing through the two objects. It is not eaſy to know in all caſes when the octant is exactly in that plane. One neceſſary caution is to make the union of the two objects (or, as we may more properly call it, the union of direct and reflected images) in the proper part of the horizon glaſs, and then, if a tube of conſiderable length be uſed to direct the ſight, the poſition of the octant cannot be very faulty. Now the union of the direct and reflected images in the horizon glaſs, ought to be made in a line parallel to the plane of the octant, but paſſing through that point of the glaſs to which the axis of the tube is directed. If a teleſcope be uſed, this union ſhould be made in a line drawn croſs the middle of the field parallel to the plane of the octant. And a teleſcope is of uſe not only to diſtinguiſh the contact or union of the images more perfectly, but likewiſe to find by the place of this union in the field, whether the octant be held in the proper plane.

109. Such a ſtand as was mentioned in par. 31 is likewiſe of ſervice for this purpoſe. Let the plane of the octant be ſet perpendicular to the axis Q Q Fig. 4, and be fixed by the ſcrews T T. Let the whole be turned round the axis Q Q; if you then ſee both objects ſucceſſively through the tube or teleſcope by direct viſion, the octant muſt of courſe be in the proper plane

110. Making the octant vibrate round the axis of viſion (or any other axis parallel to it) affords a way of knowing when the octant is held properly, as will be explained hereafter.

111. Another method peculiar to the back obſervation, but very uſeful in taking horizontal angles at land, is to obſerve whether the part of the horizon ſeen by reflection coincides with, or is parallel to that part of the horizon which is ſeen directly. If the two horizons croſs each other, the plane of the octant is not level. A ſimilar circumſtance, and equally uſeful at ſea, will be taken notice of in its proper place.

112. In order to judge of the importance of holding the octant in the proper plane, take off the ſight-vane, place the eye either above or below its proper height (to which the ſight-vanes guide it) ſo as to look at the middle of the horizon glaſs in a direction oblique to the plane of the octant. By inclining

inclining the octant, and moving the index, you may cauſe the images to be united in the middle of the horizon glaſs, notwithſtanding you look obliquely. Fix the index and note the angle. Put on the ſight-vane; and holding the octant in the proper plane, you will find the images will not now be united. Bring now the images together, and again note the angle, and you will find that the angle in the former caſe will exceed the angle in the latter by many minutes, in ſome caſes by a whole degree. The ſame may be done with the ſight vane in its place, by looking obliquely upwards, ſo as to get the union of the images to the top of the horizon glaſs; provided one of the objects be bright enough to be ſeen by reflection from the tranſparent part of the horizon glaſs.

Of the vibratory motion to be given to the octant at the time of obſervation.

113. When the images are brought nearly into contact, it is uſeful to ſwing the octant to and fro round the axis of viſion, or ſome line parallel to it. If you hold the octant as before directed, (par. 107,) you may eaſily turn it about between the fingers and thumb of the right hand. By this means the reflected image will be made to vibrate backwards and forwards acroſs the plane of the inſtrument. It is often eaſier to diſtinguiſh whether the images coincide in paſſing one another, than by bringing them together while the octant is held ſteady. Thus, in taking the diſtance of two ſtars, this motion of the reflected ſtar will make on the eye the impreſſion of a line of bright fire, which impreſſion will remain ſome time. It is eaſy to judge whether this line paſſes through or at a diſtance from the other ſtar ſeen directly.

114. In taking diſtances of the ſun or moon, from the ſea horizon, from the ſtars, or from each other, this vibratory motion of the reflected image of the ſun or moon ſerves to diſtinguiſh that part of their limb which is the neareſt or remoteſt from the other object; it is always the diſtance of one of theſe two points which is to be meaſured. This will be further explained when we treat of theſe obſervations in particular.

115. Laſtly, this vibratory motion is of ſome uſe to find whether the octant be held in the proper plane. The traces of the arch, deſcribed by the reflected image in its vibratory motion, will remain in the mind even when the octant is at reſt; and this arch ought to be perpendicular to the plane of the octant, when it is in that poſition in which the images appear united, or paſs by each other in their vibratory motion.

116. In ſtrictneſs the two images cannot appear to coincide or be united perfectly, unleſs one of the objects be bright enough to be ſeen by *reflection from* the tranſparent part of the horizon glaſs, while the other object is ſeen *directly through* the very ſame point of that glaſs. When this is not the caſe, all that can be done is to make the two images meet, and, as is were, join at the ſeparation of the ſilvered and tranſparent parts of the horizon glaſs. If the octant be turned about to and fro round the axis of viſion, ſo as to cauſe a vibratory motion of the reflected image, it will then be eaſy to judge whether the line, which that image deſcribes, would, if produced, go through the object ſeen by direct viſion, or paſs at a diſtance.

Aſtronomical Obſervations at SEA.

To take the Altitude of the ſun at ſea by the fore obſervation.

117. The ſun is generally bright enough to be ſeen by reflection from the unſilvered part of the horizon glaſs; uncover therefore the uppermoſt hole in the ſight-vane, or, if the octant is furniſhed either with a tube or a teleſcope, raiſe it ſo as to be oppoſite to the tranſparent part of the horizon glaſs; put likewiſe the red glaſſes into their proper place. Hold now the octant in a vertical poſition, and, turning yourſelf toward the ſun, direct the ſight to that part of the ſea horizon which is under the ſun: then begin to move the index from o on the limb till you ſee the red image

image of the ſun come down towards the horizon. If you meet with any difficulty in finding the red reflected image of the ſun, turn back the red glaſs, and the ſun's image will then be ſo bright that you cannot miſs it: when found, return the red glaſs into its proper ſituation. Having brought the ſun's image almoſt down to the horizon, then, with your left hand, ſwing the octant backwards and forwards, turning it round the line paſſing between the fingers and thumb of the right hand, and the image of the ſun will ſeem to deſcribe the arch of a circle whoſe convex ſide is downwards. Move the index till the lower edge of the ſun's image juſt grazes upon the horizon, when it is at the loweſt point of the arch apparently deſcribed, and then ſtop the index. Read off the degrees and minutes ſhewn by the index, and call this *the obſerved altitude*, to which the following corrections muſt be applied.

118. *Firſt.* The index error. — To be added or ſubtracted by directions in par. 54.

119. *Secondly.* The dip of the ſea horizon. — To be found in the tables belonging to the *Nautical Ephemeris*, page 14.* This is to be ſubtracted.

120. *Thirdly.* The ſun's ſemi-diameter. — This is to be found in the third page belonging to every month in the Nautical Ephemeris, and is to be added.

Theſe three corrections properly made, you will have *the apparent altitude of the ſun's center*. To this apply two other corrections, viz.

121. *Fourthly.* The refraction. — To be found in the aforeſaid tables, page 2, and is to be ſubtracted.

122. *Fifthly.* The ſun's parallax. — To be found in the additions to the Nautical Ephemeris for 1772, page 37, and is to be added.

And thus you will have *The* TRUE ALTITUDE *of the ſun's center*, (as ſeen from the center of the earth,) whence you may calculate either the hour of the day or the latitude of the place.

123. When great nicety is not required, you may ſuppoſe the ſun's ſemi-diameter to be always 16 minutes. And the laſt correction (viz. for the ſun's parallax) is ſo ſmall that it may generally be omitted.

* Publiſhed by order of the Commiſſioners of Longitude.

To take the altitude of the ſun by the back obſervation.

124. Having removed the red glaſſes into their proper place, hold the octant in your left hand, and in a vertical poſition. Direct the ſight to the part of the horizon oppoſite to the ſun: then, applying your right hand to the index, move the index gradually from 0, till you bring the red image of the ſun *up* to the horizon, ſo that the *apparent* lower edge may ſtand upon the horizon.* If you now ſwing the octant between the fingers and thumb of your left hand, the image of the ſun will deſcribe an arch of a circle whoſe convex ſide is downwards. Move the index till the apparent lower edge of the ſun's image juſt grazes upon the horizon in the loweſt point of that arch, and then read off the degrees and minutes as before; ſee par. 25. Let this be called the *obſerved altitude*, to which apply the following corrections.

125. *Firſt.* The index error. — To be added or ſubtracted according to the directions in par. 83.

126. *Secondly.* The dip of the horizon.—To be found as before, par. 119, but muſt now be added.

127. *Thirdly.* The ſun's ſemi-diameter.—To be found as before, par. 120, but muſt now be ſubtracted.

Theſe three corrections give the *apparent altitude* of the ſun's center, to which apply the other two corrections as before, par 121 and 122, and you will have the *true altitude* of the ſun's center.

128. Inſtead of making the octant vibrate between the finger and thumb of the left hand, you may uſe this motion. Keep the octant ſteady in your hands, but turn yourſelf round upon your heel; or rather, keeping your body upright, twiſt yourſelf from the right to the left, and back again, ſo as to turn the octant round a vertical axis. Obſerve whether the image of the ſun continually touches the horizon during this motion; if it does, then the plane of the octant is held exactly vertical: if, in that motion, the ſun's image croſſes the horizon, the plane of the octant is not vertical, and you muſt ſhift its poſition till you can make

* The apparent lower edge is not the real lower edge. See par. 80 and 101.

make the limb of the ſun conſtantly glide along the horizon while you turn yourſelf round as before.

Either of theſe motions ſerve as a proof that the octant is held in the proper poſition juſt at the very time when the index was ſet. The latter is that generally recommended.

To take the altitude of the moon.

129. The moon's altitude may be taken either by the fore or back obſervation, exactly in the ſame manner as the ſun's altitude, only here you muſt bring that edge of the moon into contact with the horizon, which is round and well defined, whether that be the upper or under edge. The corrections to be applied to the *obſerved altitude* are as follow:

130. *Firſt.* The index error. To be applied as before directed, par. 54, for the fore obſervation, and par. 83, for the back obſervation.

131. *Secondly.* The dip. To be found as before, par. 119, and ſubtracted in the fore obſervation, but added in the back obſervation.

132. *Thirdly.* Semidiameter. To be found in the Nautical Ephemeris for every noon and midnight at Greenwich. If very great accuracy is required, this ſemidiameter muſt be corrected, for the intermediate time and for the altitude, by the rule in page 152 of the tables belonging to the Nautical Ephemeris. The ſemidiameter, ſo found, is to be added in the fore obſervation, and ſubtracted in the back obſervation, if the moon's apparent lower limb was obſerved; that is, if the lower edge of the moon's image, as ſeen by reflection in the octant, (through plain ſights,) was brought into contact with the horizon; but if the upper edge (as ſeen by reflection in the octant) was brought into contact with the horizon, then this ſemidiameter is to be ſubtracted in the fore obſervation, and added in the back obſervation: and thus you will have the *apparent altitude* of the moon's center.

133. *Fourthly.* Refraction. To be found and applied as before, par. 121.

134.

134. *Fifthly.* Parallax. The moon's *horizontal* parallax for every noon and midnight, at Greenwich, is to be found in the Nautical Ephemeris. When nicety is required, this muſt be corrected, for all intermediate times, by the rule in page 152 of the tables belonging to the Ephemeris. The moon's *horizontal* parallax being thus obtained, the moon's parallax *in altitude* may be found therefrom by page 3 of the tables aforeſaid, and is to be added. Otherwiſe in page 52 you will find a table which gives the correction both for refraction and parallax in one. This added to the apparent gives the true altitude.

To take the altitude of a ſtar by the fore obſervation.

135. Set the index at 0; and, holding the plane of the octant vertical, direct the ſight to the ſtar, and at the ſame time look for the reflected image of the ſtar in the ſilvered part of the horizon glaſs. Stir the index a little, which will ſeparate the reflected image from the direct image; the former will be eaſily diſtinguiſhed from the latter by its motion when you ſtir the index. Continue to advance the index, and at the ſame time follow the reflected image of the ſtar with your eye, directing the ſight lower and lower, and changing the poſition of the octant as the image of the ſtar deſcends, till you have brought it down to the horizon: the index will then ſhew the obſerved altitude of the ſtar. It is neceſſary that you ſhould always keep ſight of the reflected image of the ſtar, that you may not miſtake the reflected image of ſome other ſtar for the image of that whoſe altitude you would take: but, if you know its altitude nearly, you may then ſet the index at firſt to that altitude; and, holding the octant vertical, direct the ſight to the part of the horizon juſt under the ſtar, and you will be ſure to find its reflected image near the horizon. By moving the index a little, you may bring it to touch the horizon as before.

136. The corrections to be applied to the obſerved altitude of a ſtar, are, *Firſt*, the index error, as in par. 54. *Secondly*, the dip, as in par. 119. Theſe two give the apparent altitude. *Thirdly*, the refraction, as in par. 121, which gives the true altitude. The fixed ſtars have neither ſemidiameter nor parallax.

To

To take the altitude of a ſtar by the back obſervation.

137. In this caſe you muſt know the altitude nearly, or it will be very difficult to find the reflected image of the ſtar. This known, ſet the index to the ſuppoſed altitude; hold the octant vertical; direct the ſight to the part of the horizon oppoſite to the ſtar. When you have found the reflected image, bring it to touch the horizon, by moving the index as before.

138. The altitude, ſo obſerved, muſt be corrected; *Firſt*, for the index error, by par. 83. *Secondly*, for the dip, by par. 126. *Thirdly*, for refraction, by par. 121.

The ſea horizon, I believe, can rarely be ſeen at night, unleſs, perhaps, by the twilight or moonlight, ſo that altitudes of the ſtars, ſo obſerved, cannot be much relied on.

To take the diſtance of the moon from the ſun, in order to find the longitude at ſea.

139. In the Nautical Ephemeris you have the central diſtances of the ſun and moon for every three hours of time at Greenwich, on ſuch days as this method is practicable. From theſe diſtances, in the Ephemeris, you are to compute roughly the diſtance between their neareſt limbs at the time of obſervation, as directed in page 165 and 166 of the Ephemeris. If that diſtance be under 90 degrees, you muſt uſe the fore obſervation; if above 90 degrees, the back obſervation.

140. The moon muſt be viewed directly through the unſilvered part of the horizon glaſs; but the ſun by reflection; and, if it be bright, by reflection from the unſilvered part of the glaſs. Hence, if the ſun be to the left hand of the moon, the octant muſt be held with its face downward in the fore obſervation, but with its face upwards in the back obſervation. If the ſun be to the right hand of the moon, the octant muſt be held with its face upwards in the fore obſervation, but downwards in the

the back obſervation.* One or both of the red glaſſes muſt be let down, as the ſun is more or leſs bright.

141. Set the index to the diſtance between the neareſt limbs of the ſun and moon, computed roughly as before; and, placing the face of the octant by the foregoing rules, direct the ſight to the moon. Then put the plane of the octant into *the plane of the great circle paſſing through the ſun and moon, according to par.* 108; that is, put it into ſuch a poſition, that if you look along the ſurface, or rather edgewiſe againſt the octant, it may ſeem to make a line paſſing through the ſun and moon. The knack of doing this is to be got by practice only. Swing now the octant between the finger and thumb, and the red image of the ſun will paſs by the moon to and fro, and ſo near that you cannot miſs ſeeing it. Their neareſt edges muſt then be brought into exact contact. And here obſerve, *Firſt*, that the images of the ſun and moon muſt touch only at their external edges; the body of the ſun muſt not (in this vibratory motion) paſs over, or even be upon, the body of the moon. *Secondly*, the edge of the ſun muſt touch the enlightened edge or round part of the moon, not the part from the ſun, which is either hollow or gibbous, and always ill-defined.† Theſe two cautions obſerved, move the index till (by giving the octant a vibratory motion as before) the edge of the ſun juſt bruſhes the edge of the moon as they paſs by each other. The index then ſhews, on the limb, the *obſerved diſtance* of the neareſt edges of the ſun and moon, to which apply the following corrections.

142. *Firſt*, The index error, as in par. 54 and 65.

143.

* When an octant is furniſhed with a ball and ſocket, to be fixed either againſt the face or the back of the octant, it may then be held conveniently either way. When that is not the caſe, it may be worth while to have a *left handed* octant made; that is, one in which all the parts are reverſed as to right and left. Such an one muſt be held with the face upwards, when a common one is to be held with the face downwards.

† From new moon to the full, the enlightened edge of the moon is turned toward the Weſt. From the full moon to the new, the enlightened edge is turned towards the Eaſt. This is the caſe when the moon is ſeen directly; but the contrary if it be viewed through a teleſcope that inverts the objects.

143. *Secondly.* Semidiameter. The ſemidiameter of the ſun for every ſixth day, and of the moon for every noon and midnight, at Greenwich, are to be found in the Nautical Ephemeris; whence their ſemidiameters, at the time of obſervation, are to be computed by the rule in page 152 of the tables belonging to that Ephemeris. The ſum of theſe two ſemidiameters is to be added.

Theſe two corrections, properly applied, give the *apparent diſtance* of the centers of the ſun and moon.

144. *Thirdly.* This apparent diſtance muſt be corrected for refraction and parallax, and then you will have the REDUCED DISTANCE, or TRUE DISTANCE, *of the centers of the ſun and moon, as ſeen from the center of the earth.* To make this laſt correction, two ſets of tables, with directions how to uſe them, are given among the tables belonging to the Nautical Ephemeris. The former ſet of tables (by *Mr. Lyons*) gives the corrections for refraction and parallax ſeparately. The latter (by *Mr. Dunthorne*) gives both correction in one. A ſet of rules for calculating this correction is given (by *Mr. Witchell*) in the Nautical Ephemeris for 1772. In this Ephemeris are likewiſe ſome corrections of the former tables, and additions to the directions before given, which ought not to be neglected. Laſtly, a ſet of tables for finding this correction by inſpection only, is now in the preſs, at Cambridge, under the direction of one of the profeſſors of aſtronomy, and may be expected ſhortly by the public.

145. From the reduced diſtance and time of obſervation, the LONGITUDE of the place may be computed by the rules and tables belonging to the Nautical Ephemeris.

To take the diſtance of the moon from ſuch ſtars as are ſelected in the Nautical Ephemeris, for finding the Longitude at ſea.

146. The central diſtances of theſe ſtars (like thoſe of the ſun) are given in the Nautical Ephemeris for every three hours of time at Greenwich, from whence their diſtance from the enlightened edge of the moon may be rough-

ly computed as before. None of these distances exceed 90 degrees, and therefore the fore observation only is to be used.

147. The star must be viewed directly through the transparent part of the horizon glass, but the moon by reflection: therefore, when the star is to the left hand of the moon, the octant must be held with its face upwards; when the star is to the right hand of the moon, the octant must be held with its face downwards. The fainter of the red glasses must be let down, which so far takes off the glare of the moon, that the star may be seen actually *upon* her reflected image.

148. Set the index to the distance of the star from the enlightened edge of the moon, computed roughly as before, and, placing the face of the octant by the foregoing rules, direct the sight to the star. Then put the octant into the proper plane, so that, if seen edgewise, it may seem to make a line passing through the moon and star. Swing now the octant between the finger and thumb of the right hand, and the red image of the moon will pass by the star to and fro, and so near that you will see it; and this is a proof that the sight is directed to the right star. The enlightened edge of the moon (whether East or West) must then be brought into contact with the star by moving the index.* You will know this by the vibratory motion of the moon's image before mentioned; for, the star should just graze the edge, without entering at all within the body of the moon. The index will then shew the *observed distance* of the star from the enlightened edge of the moon, to which apply the following corrections.

149. *First.* Index error, as in par. 54.

150. *Secondly.* Semidiameter of the moon. To be found in the Nautical Ephemeris for every noon and midnight at Greenwich, and thence to be computed for the time of observation by the rule in page 152 of the tables belonging to that Ephemeris. After the new and before the full moon, this semidiameter is to be subtracted if the star be East, and added if the star be West, of the moon. After the full and before the new moon this semidiameter is to be added

* See the note on par. 141.

added if the ſtar be Eaſt, and ſubtracted if the ſtar be Weſt, of the moon.

Theſe two corrections give the *apparent diſtance* of the moon's center from the ſtar.

151. *Thirdly.* This apparent diſtance muſt be corrected for refraction and parallax in the ſame manner as the apparent diſtance of the ſun was corrected par. 144. This will give the *reduced* or *true diſtance* of the center of the moon from the ſtar, as ſeen from the center of the earth.

From the reduced diſtance, and time of obſervation, the *Longitude* may be computed by the rules and tables belonging to the Nautical Ephemeris.

152. It will ſave ſome trouble, and may ſerve the purpoſe of finding the reflected image of the ſun or moon in the horizon glaſs, if you only ſet the index to the central diſtance as ſet down (in the Ephemeris) for the neareſt three hours, without correcting it to the intermediate time by a rough computation, as before directed.

153. If you have any doubt whether the ſight be directed to the proper ſtar, ſet the index to the ſuppoſed diſtance as before, and then find the reflected image of the moon in the horizon glaſs, in the following manner. Having turned back the red glaſs, hold the octant in any plane paſſing through the moon, a vertical plane for inſtance; move the octant in that plane (raiſing or lowering the ſight) and you will be ſure to find the moon's image ſomewhere. Having once found the reflected image of the moon, then turn the octant round the incident ray, (continually changing the poſition of its plane,) but ſo as ſtill to keep ſight of the moon in the horizon glaſs, and you will ſee, through the tranſparent part of it, all thoſe ſtars which have the diſtance ſhewn by the index. The ſtar to be obſerved will lie in a line nearly perpendicular to the horns of the moon or her longer axis. As the ſtar is always a bright one, and the circle in which it lies may be thus traced out, there will be no danger of miſtaking it.

The directions before given (par. 139 and the following) hold exactly the ſame, whether you uſe plain ſights or a teleſcope.

154. In theſe obſervations great care muſt be taken to keep the octant in the proper plane, according to par. 108. In common octants, where the back vane ſtands near the horizon glaſs, and has only a plain hole, it is not difficult to look ſo obliquely as to make a difference in the obſerved diſtance of half a degree or more; and almoſt as great a difference may be made in the fore obſervation, even though a tube or teleſcope direct the ſight. The poſitions in which the inſtrument muſt be held, to make theſe obſervations for the Longitude, are ſo various, and ſome of them ſo uncouth, that it may be many years before ſeamen in general can take theſe diſtances as exactly as they do now take an altitude of the ſun. Nevertheleſs the great dexterity required in ſome arts daily exerciſed may convince us, that there is no manual operation in which the utmoſt accuracy may not at laſt be attained by conſtant practice.* In the Philoſoph. Tranſactions, N°. 425, the inventor of this inſtrument gives a rule for finding the error occaſioned by holding it out of the proper plane. Every ſuch rule requires a previous knowledge of the deviation from that plane, which is not eaſy to get exactly, eſpecially at ſea. It is better to truſt to the ſkill of the obſerver in holding it properly, and to ſuch a conſtruction of the inſtrument as will not permit an unſkillful perſon to ſee the object at all, in thoſe caſes where the error would be very great.

Aſtronomical Obſervations at LAND.

To obſerve the altitude of the ſun by reflection from water.

155. The water muſt be ſheltered from the action of the wind by the apparatus deſcribed in par. 33. The water-trough, *p q r s*, fig. 9, muſt be ſet upon the ground, or ſome

* Nothing ſhows this more plainly than the aſtoniſhing degree of accuracy and perfection to which the art of playing upon muſical inſtruments (of all ſorts) is now carried.

ſome firm wall, with its ſides nearly in the ſun's azimuth, which will be known by their caſting no ſhadow. The box, fig. 8, muſt then be ſet over it, but ſo as not to touch the trough. The glaſs planes muſt be ſet to the expected angle, and then put within the box ſo as to cover the trough.* The eye muſt be defended with ſuch a ſmoked glaſs as is mentioned in par. 16, and one of the light-red glaſſes let down. By this means the image of the ſun reflected from the water will appear yellow, and the image reflected from the mirrors will appear red: this is of great uſe to diſtinguiſh them from each other. The fore obſervation muſt be uſed if the ſun's altitude be leſs than 45 degrees, but the back obſervation if that altitude be greater than 45 degrees.

156. Hold now the octant in a vertical plane, and direct the ſight to the ſun's image in the water; then move the index till you ſee the other image reflected by the mirrors, which will be known by its red colour. If there ſhould be any difficulty in finding either image, put aſide the ſmoked glaſs in one caſe, and the red glaſs in the other, and you will be ſure to ſee the ſun. When you have found both images, then, if you uſe a teleſcope, make them as diſtinct as you can;† likewiſe raiſe or lower it, till both images are equally bright. See par. 16.

157. All things thus prepared, move the index ſo as to bring the edges of the two images into contact, making the octant vibrate a little to diſtinguiſh that contact more perfectly, as before directed. Obſerve now whether the red ſun or the yellow ſun be uppermoſt: for, the index will ſhew double the altitude of the ſun's upper limb, if the yellow image be uppermoſt; but double the altitude of the lower limb, if the red image be uppermoſt. This is on a ſuppoſition

* The ſun's altitude at noon can always be gueſſed near enough to ſet the glaſs planes. At other times you may make a rough obſervation firſt to find the altitude nearly, and then repeat it with the planes ſet exact. Four or five degrees in the angle of the planes is of no conſequence.

† Both images cannot be perfectly diſtinct, becauſe of the difference of their colour, but they will be very nearly ſo, if the teleſcope does not magnify too much.

ſuppoſition that you uſe plain ſights; the contrary is the rule if you uſe a teleſcope that inverts.

158. Having thus found the double altitude, apply to it the correction for the index-error by the rules in par. 54 and 65, and half of this gives the obſerved altitude of the ſun's upper or lower limb, according as the yellow or red image was uppermoſt. Here remember to correct the double altitude for the index-error before you halve it: if you will halve it firſt, then you muſt correct that half by applying only *half* the index-error, found by the rules before mentioned.

159. To this altitude you muſt apply the correction for the ſun's ſemi-diameter, to be found in the Nautical Ephemeris at the third page belonging to every month. This correction is to be added if the altitude of the lower limb was obſerved, but ſubtracted if the upper limb was obſerved, and then you have the apparent altitude of the ſun's center. This being farther corrected, for refraction and parallax, by the rules referred to in par. 121 and 122, you have the TRUE ALTITUDE of the ſun's center, whence the latitude of place or hour of the day may be derived.*

160. If there be any ſuſpicion that the glaſs planes are not ground true, repeat the obſervation, but change their places, ſo that the plane, which before received the ſun's direct rays, ſhall now receive the rays reflected from the water, and contrariwiſe; for, the refraction of the planes will now have a contrary effect, and, if it before increaſed the obſerved altitude, will now diminiſh it. By thus changing the poſition of the planes, and arguing from a number of obſervations, either the latitude of the place or the hour of the day may be got very near the truth.

161. If the octant be held in the hand only, it will be neceſſary to reſt the arms on ſomewhat, and likewiſe to ſet the limb of the octant on a ſupport; ſo that one hand may keep the octant in the ſun's azimuth, (holding it as directed in par. 107,) while the other governs the index. But, all this trouble may be avoided by mounting the octant

* See an example of determining the latitude by this inſtrument in the Phil. Tranſ. Vol. LIX. page 239.

tant on a ſtand, as before ſuggeſted, and then the obſervation may be made with eaſe and accuracy.

162. Some recommend treacle, or treacle diluted with water, to make the reflection. But, if the fluid be ſo thick and tenacious that its ſurface cannot be diſturbed by the wind, then there will be much uncertainty whether it be level; and, if the liquor be ſo much diluted as to put this out of doubt, then the leaſt breath of wind will raiſe tremors; ſo that glaſs planes, though expenſive, are, I fear, neceſſary.

To obſerve the altitude of a ſtar.

163. The proceſs is exactly the ſame as before, only here you muſt uſe quickſilver, the reflection from water not being bright enough, unleſs for a few ſtars of the firſt magnitude. The quickſilver is very apt to collect duſt on its ſurface; it ſhould be ſtrained through a ſilk handkerchief juſt before the obſervation.

164. The double altitude of the ſtar muſt be firſt corrected for the index-error by par. 54 or 65, and then halved. That half, being corrected for refraction as before, par. 121, gives the true altitude.

To obſerve correſponding altitudes of the ſun.

165. This is one of the moſt uſeful operations in practical aſtronomy. You may by this means examine the going of a clock, and determine its error without a meridian line. The exact time of any appearance in the heavens is always a capital circumſtance in aſtronomical obſervations. A method of determining that time, without the apparatus of a fixed obſervatory, is therefore very uſeful, eſpecially to thoſe who go abroad.* Correſponding altitudes are likewiſe very uſeful in finding a meridian line, or adjuſting the poſition of a tranſit teleſcope. Theſe obſervations

* If a traveller takes with him this octant, a ſmall teleſcope, and a portable time-piece (a ſpring-clock for inſtance) to keep time for a day or two only, he may make very uſeful obſervations without more expenſive and cumberſome inſtruments.

ſervations are not only uſeful but eaſy to make with almoſt any octant; for, they require no rectification of the glaſſes, no accuracy in the diviſions on the limb, nor any great truth in the planes that ſhade the water.

166. Theſe altitudes ſhould be obſerved [in our latitude] at leaſt two hours diſtant from noon, and therefore the fore horizon glaſs is always to be uſed. The beſt time is when the ſun is on or near the *Prime Vertical*, that is, the Eaſt or Weſt points of the compaſs.

167. Having prepared all things in the morning as before mentioned, par. 155, direct the ſight (which we will here ſuppoſe to be the plain tube) to the ſun's image in the water: move the index till you ſee the image reflected by the mirrors, or the red ſun: bring down this red ſun (by moving the index) a little below the yellow ſun, and fix the index by the ſetting ſcrew.

168. Wait now till the upper limb of the red ſun, by riſing, juſt touches the lower limb of the yellow ſun, making the red image now and then vibrate acroſs the yellow one, to be ſure of the contact of their neareſt edges; then note the hour, minute, and ſecond, ſhewn by the clock, either by calling to your aſſiſtant at the clock at the inſtant you ſee the contact of the images, or by cauſing him to count the ſeconds aloud.

169. Again wait till the red ſun, by riſing, juſt covers the yellow ſun, which may be eaſily known as the images are of two colours, eſpecially if you now and then make the red image vibrate ſidewiſe as before directed. Note the time by the clock as before.

170. Laſtly, wait till the lower edge of the red ſun juſt touches the upper edge of the yellow ſun, note the time by the clock, and you have finiſhed one *ſet* of morning obſervations. Enter theſe down in one column, under the title of *Eaſtern Azimuth*. Examine now the degrees and minutes ſhewn by the index, and enter it at the head of the whole, under the title of *Double Altitude*.

171. If you chooſe to make more than one ſet of obſervations in that day, then releaſe the index, and, ſetting it forward, repeat the former work, always remembering to

leave

leave the index fixed when the laſt ſet of morning obſervations is completed.

172. Near the correſponding time in the afternoon (at which you may make a rough gueſs) prepare the water apparatus as before, par 155, and be careful to ſet the glaſs planes, that cover the water, in the ſame poſition as before in the morning; that is, turn the ſame plane towards the ſun as before. Take now the octant with the index yet fixed, and direct the ſight to the ſun's image in the water, and you will find the red ſun uppermoſt, but may eaſily perceive it to ſink ſo as to approach the yellow ſun. Wait for the contact of the lower edge of the red ſun with the upper edge of the yellow one, and note the time ſhewn by the clock as before. In like manner note the time of the coincidence of the two images, alſo the time of the contact of the upper edge of the red ſun with the lower edge of the yellow one; and you have one complete ſet (of the times) of correſponding altitudes of the ſun.

173. Releaſe now the index, and ſet it carefully to the degree and minute at which it before ſtood during the ſet of morning obſervations preceding this laſt, and then you are prepared for the afternoon obſervations correſponding to that ſet, and which come on next; carry theſe on as before, and ſo till every ſet is completed.

174. Enter the times of the afternoon obſervations in the ſame line of your paper with the morning obſervations to which they correſpond, but in a different column, under the title of *Weſtern Azimuth*. Thus, when you begin in the morning, enter the ſucceſſive times on the left hand, one below another, deſcending as you proceed. In the afternoon begin on the right hand of theſe at the bottom line, aſcending as you proceed; an example of which you may ſee in the Philoſophical Tranſactions, Vol. LIX. page 237, and Vol. LX. page 357.

175. Take now the firſt pair of correſponding times and add them together; halve their ſum, to which add ſix hours, and it gives the time of ſolar noon derived from this pair; therefore ſet it down on the ſame line with this pair; but, in another column, under the title of *Meridian*. Do the ſame by the other two pairs in the firſt

ſet. Take the mean of theſe three times of ſolar noon, correct it for the change of the ſun's declination between the morning and afternoon obſervations,* and you have the time, ſhewn by the clock at ſolar noon, inferred from the firſt ſet of obſervations. In the ſame manner may that time be inferred from each ſeparate ſet. Take the mean of all the times thus found from each ſeparate ſet, and you will have the exact time ſhewn by the clock at ſolar noon; that is, when the center of the ſun is upon the meridian. Look now in the Nautical Ephemeris for the equation of time, which you will find in the ſecond page belonging to every month. Add or ſubtract this from twelve hours, (according to the title at the head of the column in the Ephemeris,) and you have the true time of ſolar noon, or the time that would be ſhewn, when the ſun is upon the meridian, by a clock that went perfectly right. Compare this with the time actually ſhewn at the ſame inſtant by your own clock, (found as before,) and you will have its error at the ſolar noon of that day with great exactneſs.

176. If the different ſets of obſervations, made on the ſame day, follow each other within an hour of the equinoxes, or within two near the ſolſtices, you need not correct each ſet ſeparately. It will be ſufficient to take one mean of the whole and correct that as before.

177. If you uſe a teleſcope that inverts, the motions of the ſun will be contrary to thoſe here deſcribed. Thus the red ſun will fall in the morning and riſe in the afternoon.

178. Correſponding altitudes of the principal ſtars may be taken in like manner by reflection from quickſilver. You may, in the ſame night, take ſeveral obſervations of ſome one capital ſtar at different altitudes, removing the index for each obſervation; or, which is better, keep the index always fixed, and take correſponding obſervations

* A table, giving this correction for the latitude of London, and for every ſix degrees of the ſun's longitude, is in the *Aſtronomical Obſervations*, &c. printed for *T. Cadell*. A table, whence this correction may be eaſily computed for all latitudes, is given in the Nautical Ephemeris for 1769.

tions of different ſtars as they arrive ſucceſſively at the ſame altitude. No correction for change of declination is wanted in this caſe. The time of a ſtar's paſſage over the meridian will be preciſely equidiſtant from the times of its attaining an equal altitude on the Eaſtern and Weſtern azimuths.

Of taking diſtances of the ſtars from each other.

179. Diſtances of the planets or fixed ſtars from each other may eaſily be taken by the octant, if mounted on ſuch a ſtand as is deſcribed in the *Aſtronomical Obſervations*, &c. After the Octant is got into the proper poſition, (by the directions given in that book,) ſet the index to 0, and then direct the ſight to the brighter of the two ſtars. Stir the index a little to get ſight of the reflected image of that ſtar. Continue to advance the index; and at the ſame time, follow that image with your eye, turning the octant round the axis QQ (plate the firſt of that book) till the reflected image is arrived at the other ſtar, which you will ſee by direct viſion Make the two ſtars coincide, as directed par. 113, and the index will ſhew on the limb, their obſerved diſtance. This proceſs reſembles that more particularly deſcribed in par. 135.

180. Diſtances ſo obſerved may be very uſeful to determine the orbit of a comet. The motion of comets is ſuch that they can ſeldom be ſeen on the meridian; and when they cannot, this is the beſt way of determining their places in the heavens. It were to be wiſhed that the uſe of this inſtrument in taking ſuch diſtances was every where known. The ſtay of thoſe extraordinary bodies in our regions is often ſo ſhort, that, if the ſky be unfavourable a few nights at our great obſervatories, we can learn nothing ſatisfactory about them; thoſe who chance to ſee them at other places not knowing how to aſcertain the circumſtances upon which the inveſtigation of their motions muſt be founded. Theſe diſtances from the fixed ſtars muſt be corrected for index error and refraction. The obſerver muſt carefully note the former; aſtronomers will ſupply the latter, if either the altitude of the ſtars or the

hour of the night be known. Thoſe who would calculate the effect of refraction upon the diſtance, may conſult the tables belonging to the Nautical Ephemeris, page 19.

Topographical Obſervations, &c.

The uſe of the octant in county ſurveys.

181. The octant is of ſome uſe in taking the angles between diſtant objects on the horizon. Dark objects againſt a bright ſky, or bright objects againſt a dark ſky, are eaſily diſtinguiſhed even by the naked eye, and the coincidence of their images in the octant may be well aſcertained. But objects below the horizon can hardly be ſeen at all in the octant, eſpecially if ſmall or diſtant. The objects ſeen directly loſe much of their light by its being tranſmitted through the upper part of the horizon glaſs, which had better be cut off, as was before obſerved, par. 13.

182. It is proper to take the diſtance of every object from two or three others, and likewiſe the diſtance of theſe laſt mutually. Thus take the diſtance of *A* from *B* and *C*, then the diſtance of *B* from *C* and *D*, &c. Thus the obſervations will check each other. Not that the ſum of the diſtances of ſeveral intermediate objects will always be equal to the diſtance of the extremes; or that the ſum of the angles taken round the horizon will always make 360 degrees. This is not to be expected, be the obſervations ever ſo exact, unleſs the objects all ſtood in one plane. Yet the method before mentioned is a good check againſt groſs errors, and particularly againſt miſtakes in reading off the angles. Theſe diſtances need no other correction but for index error. If, indeed, you could take the elevation or depreſſion of every object, you might farther correct the obſervations on that account, but it would greatly increaſe the work, and this correction for diſtant objects is ſo ſmall that it may well be omitted.

183.

183. Although the octant will not take angles between distant objects with all the accuracy that astronomers require, yet it does the work of a common survey moderately well, and very expeditiously. And, if a more exact survey was to be taken with a better instrument, yet, even then, the octant would be of great use to make a rough survey first, in order to judge where the *principal stations* should be fixed. There ought to be five or six of these stations in every good survey, so chosen as to be every one in sight of all the rest, and such that you may command any part of the county from (at least) two of them. The places of these principal stations may be laid down by a larger instrument furnished with telescopic sights. The octant may then do very well for inserting villages and places of less note, whose distances will probably be near some or other of the principal stations.

184. It is often useful in these surveys to continue a right line quite through a county. To do this, we must determine where a line, drawn from a given point in the horizon through the eye of the surveyor, will again cut the horizon. This is no other than to set off an angle of 180 degrees from a given object. Set the index to 180 degrees (according to the back observation, par. 24;) and, applying the eye to the back vane, direct the sight to the given object. Note what point of the horizon, seen by reflection, coincides with the proposed object seen directly, for, that is the point where the line again cuts the horizon. Otherwise you may direct the sight to the part of the horizon nearly opposite to the given object, and then observe what point seen directly coincides with that object now seen by reflection. One or other of these methods is preferable, as the proposed object, or the part of the horizon opposite to it, is more or less bright.* By this means you may

* In strictness, a line, drawn from the object seen by reflection to the center of the index glass, is in this case parallel to a line drawn from the object seen directly through the center of the horizon glass, or through the eye of the observer. These two lines therefore do not make one and the same line continued: but, as their perpendicular distance can never be more than five or six inches, in these surveys they may be accounted as one and the same line. An error of five or six inches, in the course of a line 15 or 20 miles long, is nothing.

may trace a meridian line through a county, or even through a kingdom, as has been done in France and Italy; and, though this inſtrument will not determine that line very exactly, yet it is exceeding uſeful to find by this means what eminences it paſſes over, that you may know where to convey more expenſive inſtruments, whoſe ſize and delicacy do not admit of their being carried about without trouble and danger.

185. It is of great uſe to a ſurveyor to know when he is in the line joining two diſtant objects. In county ſurveys it is often difficult to know what a very diſtant object is, eſpecially if it be ſeen only juſt above ſome intermediate eminence. The only ſure way to aſcertain a place ſo ſeen is to go to that intermediate eminence, whence you may view that place at a much nearer diſtance, and in the direct line: for, unleſs you ſee it in the ſame direction, the appearance will be ſo much changed that it may not be known. Here then a method of finding when you are exactly in that line is of great uſe. When therefore, you are near the intermediate place, ſet the index of the octant to 180 degrees as before; direct the ſight to any object you ſuppoſe may be that ſought, and obſerve whether it now coincides with your former ſtation ſeen by reflection: if not, change your place till it does, and then you are in the line joining your former ſtation and the object. Otherwiſe you may direct the ſight to your former ſtation and obſerve the diſtant object by reflection, juſt as one or the other is more eaſily diſcerned.*

The

* The octant will do this buſineſs very well, but I found it uſeful to have a much lighter and ſmaller inſtrument made for this purpoſe only: one that may be carried in the pocket, and uſed on horſeback. Mine (which may be called a *dactliner*) is a ſquare tube, 1½ inches in the ſide, 8½ inches long, and weighs half a pound.

In taking angles from *Bardon* hill, in *Leiceſterſhire*, I ſaw ſome very diſtant land, juſt peeping over (*Cannock*) *Cank-heath* in *Staffordſhire*. *Cank-heath* is 25 miles diſtant from *Bardon*: the land over it could be ſeen only by the light of the ſetting ſun almoſt behind it, nor then but by the aſſiſtance of a good teleſcope that magnified 20 times. Here then this inſtrument was of great uſe to find the part of *Cank-heath* over which the line paſſed that joined the ſtation at *Bardon* and the object ſeen over the Heath. In ſeeking this place it was eaſy to ſee, on horſeback,

The uſe of the octant in ſurveying land.

186. Theſe ſurveys are ſometimes carried on by taking angles, ſometimes by meaſurement only. The beſt ſurveyors uſe both methods as a check upon each other. The octant will take theſe angles more expeditiouſly and with far leſs trouble than a Theodolite, becauſe that inſtrument requires ſteadineſs: but then theſe angles will be taken in the ſeveral planes in which the objects lie, and not reduced to an horizontal plane as in the Theodolite. If the ground be very uneven, this may occaſion errors in plotting the lordſhip, and conſequently in the meaſurement taken from ſuch a plot. The ſeveral elevations and depreſſions might indeed be taken, and the angles reduced by calculation to the horizontal plane, but this correction greatly increaſes the work.

187. Beſides the forementioned corrections, another may be neceſſary if the objects are very near. For, the octant ſhews on the limb of the angle which a ray, drawn from the reflected object through the center of the index glaſs, makes with a Ray drawn from the other object through the center of the horizon glaſs, or, which is the ſame, through the ſight-vane.* Now the eye is always placed at the ſight-vane: therefore when the former of thoſe lines interſects the latter, juſt at the ſight-vane, then the octant ſhews the exact angle which the objects would ſubtend to the eye ſo placed, were the inſtrument removed. If the interſection of thoſe lines falls elſewhere, there will be a difference between

horſeback, when you approached and when you paſſed beyond this line. The place ſought, over which the line paſſed, proved to be a part of *Cank-heath* near *Style-Copſe*, about half way between *Cank* and *Litchfield*. From this place the object could be plainly ſeen. It put on the ſame appearance, and was well known in that country to be the *Wreken* in *Shropſhire*.

Bardon is by eſtimation 50 miles diſtant from the *Wreken*, which I believe is as far as any two objects can be ſeen from each other in England.

At *Bardon*, the *Wreken* lies to the right of *Litchfield*; the angle between them is three degrees, as near as it could be gueſſed from the field of the teleſcope.

* See the note at the end of par. 93, No. 1.

tween the angle ſhewn on the limb and that which the objects ſubtend to the naked eye. This difference is ſeldom worth conſidering, however the rule for finding the correction neceſſary on that account is in the *Aſtronomical Obſervations*, &c. par. 105. That rule relates to fore obſervations only; a ſimilar correction ſhould be made in back obſervations, the rule for which may be eaſily derived from what is there laid down.

In adjuſting the fore horizon glaſs, if, inſtead of a diſtant object, you adjuſt it by that object which is to be viewed directly, following in other reſpects the directions in par. 52; the octant will then ſhew, on the limb, the exact angle which the objects ſubtend at the center of the index-glaſs; and that whether they are near or diſtant. This indeed is not the angle which thoſe objects would ſubtend to the naked eye placed at the ſight-vane; but it is ſometimes uſeful to determine the angle which they ſubtend at the center of the index-glaſs.

Otherwiſe, if you have no mind to alter the adjuſtment already made by a diſtant object, then find the index error as directed in par. 54; only uſing, for this purpoſe, that object which is to be viewed directly, inſtead of a diſtant one. The index error, ſo found, muſt be added to, or ſubtracted from, the angle ſhewn on the limb by the rule in par. 54. This will give the exact angle which the two objects ſubtend at the center of the index glaſs as before.

The difference between the angle ſhewn on the limb (ſuppoſing the horizon-glaſs adjuſted by a diſtant object) and the angle which the objects ſubtend at the center of the index-glaſs, depends wholly on the diſtance of the object ſeen by direct viſion. The difference between the former angle and that which the objects ſubtend to the naked eye (at the ſight-vane) depends on the place of the object ſeen by reflection, that is on its diſtance and the angle ſhewn on the limb; not at all on the diſtance of the object ſeen directly.

188. When a ſurvey is made by meaſurement only, it is uſual to mark out a right line near every boundary fence; to this line perpendiculars are let fall from every bend in the fence. Such a line is called a *ſtation-line*, each end of it

it a *ſtation*, at which is placed a *ſtation-ſtaff*. The perpendiculars are called *off-ſets*, and are meaſured with the *off-ſet-ſtaff*. The diſtances of the ſeveral points where the off-ſets cut the ſtation-line (reckoned from the firſt ſtation) are all meaſured with the chain. A line being then drawn upon paper to repreſent the ſtation-line, the ſeveral perpendiculars are ſet off at their proper diſtances by a *plotting-ſcale*. Join the extremities of all theſe perpendiculars, and you will have a plan of the boundary ſimilar to the real one. And thus a whole Lordſhip may be plotted.

189. In theſe ſurveys it is uſeful to run a line through the middle of the Lordſhip, and perhaps another acroſs it, and then to take off-ſets from thoſe lines to any remarkable points, eſpecially in the fence that bounds the whole. This keeps the map of the whole in its juſt form. For, though the ſeveral incloſures may be ſeparately laid down near enough to aſcertain the quantity of acres they contain, yet, if ſmall errors be committed in the form of each, when the whole is ſet together, error will be heaped upon error, and the whole will by no means reſemble the true ſhape of the Lordſhip.

190. So many off-ſets being neceſſary, a method of taking them readily is valuable. Nothing can do it more expeditiouſly than this octant. Adjuſt the inſtrument and ſet the index to 90 degrees; let the ſurveyor walk along the ſtation line with the octant in his hand, always directing the ſight to the farther ſtation-ſtaff: let his aſſiſtant walk along the boundary line. Then, if the ſurveyor would make an off-ſet from a given point in the ſtation-line, let him ſtop at that place and wait till he ſees his aſſiſtant by reflection in the octant, and the aſſiſtant is then at the point in the boundary through which that off-ſet paſſes. On the other hand, if the ſurveyor would take an off-ſet from a given point or bend in the boundary, let the aſſiſtant ſtop at that place, and let the ſurveyor walk on in the ſtation-line till he ſees his aſſiſtant by reflection in the octant, and that will be the point where an off-ſet from the propoſed point or bend will fall.

191. If you would have theſe points determined with great preciſion, then, upon the plane of the octant, draw

a line from the ſight-vane through the middle of the horizon glaſs: from the center of the index-glaſs draw another line, cutting the former at right angles: mark their interſection, and drill a ſmall hole through it:* from this hole ſuſpend a pointed plummet; bring this over the ſtation-line, and it will determine the place of the off-ſet to a quarter of an inch. The octant in this caſe ſhould be placed on the ſtand, deſcribed par. 31; the plumb-line will fall clear of the head of the ſtand. If you would make an off-ſet from a given point in the ſtation-line, ſet up a ſpiked ſtaff in that point; place this hole in the octant on the top of the ſpike; then, directing the ſight along the ſtation-line, you will ſee by reflection the point in the boundary-line where the off-ſet cuts it. In ſome circumſtances it may be neceſſary to turn the face of the octant downwards, but this is no inconvenience if you uſe a ſtand. By this means you may make off-ſets (that is, erect perpendiculars to a given line) in a garden, or even in a houſe, with ſufficient exactneſs for the purpoſes of building.

192. Theſe off-ſet lines are drawn, as we ſaid, at right angles to the ſtation-line; but, in ſome caſes, it may be neceſſary to draw a line to anſwer the purpoſe of an off-ſet, which ſhall make ſome other given angle with the ſtation-line: in this caſe you have nothing to do but to ſet the index to the propoſed angle, and proceed as before. The line on the ſurface of the octant, paſſing through the center of the Index-glaſs, muſt now be drawn ſo as to interſect the other line (paſſing through the ſight-vane and horizon glaſs) in the given angle; and then that interſection will determine the angular point through which the hole muſt be drilled to aſcertain the place of the off-ſet as before.

If you find the index-error as directed in par. 54; only uſing, for this purpoſe, that object which is to be viewed directly inſtead of a diſtant one, and ſet the index, ſo that the angle ſhewn on the limb, when corrected for index-error, may be the propoſed angle of the off-ſet; the angular point will then always fall on the center of the index-glaſs: by this means that point will be invariable whatever

* A ſmall bar ſhould be left in the pattern by the maker, for the ſake of this hole.

ever be the angle of the off-ſet, but then the index-error muſt be determined over again for every particular off-ſet that is to be made. Upon the whole, the method before deſcribed ſeems preferable.

To take the height of a building by reflection from water.

193. The angle of the elevation of the building may be taken by the octant in a way much reſembling that deſcribed in par. 155. The water trough muſt ſtand on the ground, and the box be ſet over it ſo as not to touch the trough: the glaſs planes muſt be ſet to the expected angle, and put within the box ſo as to cover the trough: hold now the octant in a vertical plane, direct the ſight to the image in the water, and move the index till you ſee the other image reflected by the mirrors: theſe images will be known from each other by their poſition; the image reflected by the water will be inverted, that by the mirrors erect, if you uſe plain ſights; the contrary if you uſe a teleſcope that inverts.

194. Having found both images, bring their extremities into contact, making the octant vibrate as before to aſcertain this contact; the index will then ſhew the double elevation of the building above the ſurface of the water, or double the angle it makes with the horizon, ſuppoſing the eye placed at the point of reflection from the water. This angle muſt be corrected for the index-error before you halve it.

195. The octant ſhould be held as near the water as may be to leſſen another error we are going to conſider: for, if the object be ſo near that the two points of reflection (that from the water, and that from the index glaſs) cannot be conſidered as one, a farther correction will be neceſſary. Eſtimate the diſtance of the top of the building from the water, by an obſervation taken as before: from this, and the diſtance between the water and index-glaſs at the time of obſervation, you muſt compute the angle which the two points of reflection ſubtend at the top of the building. It is not difficult to ſee exactly from what part of the water the reflection is made; but we will ſuppoſe

that reflection to be always made from the middle of the water-trough, and the other from the middle of the index-glass. Make then an estimate of the angle which the middle of the water-trough and the middle of the index-glass subtend at the top of the building when the octant is in the position in which the observation was made: add this angle to that shewn on the limb (whether by the fore or back observation*); correct the whole for index error and then halve it: this half is the exact angle which a ray, drawn from the top of the building to the point of reflection on the water, makes with the surface of the water; or it is the angle of the elevation of the building above the level of the water, supposing the eye placed at the point of reflection or middle of the water-trough: if the distance of this point from the bottom of the building be known, the height of the building above the level of the water is easily found by plane trigonometry.

POSTSCRIPT.

Of the errors which arise from not adjusting the index glass.

196. In par. 44 we have shewn the method of setting the index glass perpendicular to the plane of the octant. As some instruments have no provision for doing this, the reader may wish to be informed what errors will arise from neglecting this adjustment. Now, if the two adjustments of the horizon glass, described par. 48 and 51, be properly made, then the glasses will be parallel to each other when the index stands at the beginning of the divisions;

* In the back observation it is possible for the ray, incident on the water, to pass between the index glass and the eye: shading this interval by the hand, will, in that case, hide the image in the water, and shew whether it be so or not. This case is not probable; but, if it should happen, then this angle is to be *subtracted* from that shewn on the limb.

but

but both of them deviate (equally) from the perpendicular to the plane of the octant.

197. In this case the index will not shew, on the limb, the double inclination of the speculums; for, their common section is never perpendicular to the plane of the octant, nor to both the sections of the reflecting planes made by the plane of the octant Therefore the angle between these last-mentioned sections, equal to half the angle shewn by the index, is not that measure of the inclination of the reflecting planes to each other, which is laid down by *Euclid* in the sixth definition of the eleventh book of the *Elements*; nor does the index shew, as it ought the double of their inclination.. To correct this, say, as radius is to the cosine of deviation, so is the sine of one fourth of the angle shewn on the limb, to the sine of one fourth of the corrected angle; which is, therefore, always less than those shewn on the limb.

198. But, besides this, another and greater error will arise if you look through a tube or telescope whose axis is parallel to the plane of the octant. All observations should be made in a plane perpendicular to both the speculums, and then the objects may be considered (according to Hadley's method, Phil. Transf. Nr 420 and 425) as lying in the circumference of a great circle in the heavens, whose plane is perpendicular to the common section of the speculums. For this end, the telescope should be in that plane; but, in the case we have supposed, the telescope is inclined to that plane. Therefore the objects must here be considered (in Hadley's way) as lying in a parallel circle, whose distance from the great circle is the angle which the telescope makes with that plane; or, which is the same thing, the complement of that distance is equal to the angle which a line, drawn through the common section of the speculums, parallel to the axis of the telescope, makes with that common section.

199. To find this angle, besides the angle of the deviation of the glasses and the angle shewn by the index, we must likewise know the angle which a line, drawn through the common section of the plane of the octant and horizon

horizon glaſs, parallel to the axis of the teleſcope, makes with that ſection. This will be nearly the complement of the angle of incidence on the horizon glaſs, and may be ſuppoſed equal to it. From theſe data we may compute the diſtance of the parallel of the great circle before mentioned, and thence, by (Hadley's fifth corollary) the true angle between the objects.

200. An example may explain this. Let the angle of deviation of the glaſſes from the perpendicular be one degree; the angle ſhewn by the index on the limb, 90 degrees; and the third angle before mentioned, 74 degrees and 30 minutes; being the complement of the angle of incidence given in par. 95.

	D	M	S
Then the corrected angle, or double inclination of the reflecting planes to each other, is — — — — —	89	59	8
The inclination of their common ſection to the plane of the octant — — — — — — — — — —	88	55	$3\frac{1}{2}$
The complement of the diſtance of the parallel from the great circle — — — — — — — — —	88	55	$32\frac{1}{2}$
The true angle between the objects — — — — —	89	57	$55\frac{1}{2}$

Hence the error from the firſt cauſe, par. 197, is 52 ſeconds; from the latter, par. 198, one minute and $12\frac{1}{2}$ ſeconds; from both together ſomewhat above two minutes.

THE

THEORY

OF

HADLEY's QUADRANT,

OR

The RULES for the CONSTRUCTION

AND

USE of that INSTRUMENT demonſtrated.

By the Reverend Mr. *LUDLAM.*

THE

THEORY

OF

HADLEY's QUADRANT, &c.

1. LET *NIn*, and *RHr*. fig. 1, repreſent the ſections of the plane of the figure by the ſurfaces of two ſpeculums ſtanding perpendicularly thereon. Let *BI* be a ray of light falling on a point *I* of the firſt ſpeculum, and reflected thence to a point *H* of the ſecond ſpeculum, and by it into the line *HO*: then the angle between the incident ray, and the ray after two reflections, is double the angle between the ſpeculums.

2. Since the incident ray *BI* is in the plane of the figure, the reflected ray will be ſo too;* and becauſe the ray *IH* is in that plane, ſo will alſo the reflected ray *HO*;* the rays *BI* and *HO* therefore being both in the plane of the figure, let them (when produced) meet in *E*, and let the two ſections *NIn* and *RHr* (produced) meet in *S*, then will *ISH* be the inclination of the reflecting planes to each other; † but *IHS* is the difference of the angles *IHR* and *HIS*.‡ Produce *EH* backwards to *A*, and becauſe by the laws of reflection ‖ *IHR*=*EHS*=*RHA*§, therefore *IHA* is double of *IHR*. In like manner becauſe ‖ *HIS*=*BIN*=*SIE*§, therefore *HIE* is double *HIS*; but *IEH* is the difference between *IHA* and *HIE*¶, that is between twice *IHR* and twice *HIS*, or it is double the difference of the angles *IHR* and *HIS*, that is double the angle *ISH*.

* *Smith*'s Optics, Art. 7. † Def. 6, and Prop. 19. El. XI.
‡ 32 El. I. ‖ Smith's Optics, Art. 9. § 15 El. I.
¶ Def. 6. Prop. 19. El. XI.

3. This

3. This demonſtration is general, but the figure repreſents the diſpoſition of the glaſſes and courſe of the rays in the fore obſervation. It may be more convincing, and help us to clearer notions, to ſee it particularly applied to the back obſervation Fig. 2, then repreſents the diſpoſition of the glaſſes and courſe of the rays in the back obſervation. In this let the two ſections *NIn* and *RHr*, as before meet in *S*. Produce *HS* to *T*, and *IST* is the inclination of the reflecting planes to each other.* Let *BI* the incident ray, and *HO* the ray after two reflections, (when produced,) meet in *E*; and produce *OHE* to *A*, ſo that *IEA* may be the *obtuſe* angle which theſe rays make with each other. Then is *IEA* the ſum of *IHE* and *HIE*†, the former of which (by the laws of reflection‡) is double of *IHS*, and the latter double of *HIS*. Therefore *IEA* is double the ſum of *IHS* and *HIS*; that is double of *IST*.†

4. Let *SH* and *SI*, fig. 3, repreſent the ſections of the plane of the figure by the ſurfaces of two ſpeculums ſtanding perpendicularly thereon. Let theſe ſections meet in *S*, and *ISH* will be the inclination of the reflecting planes and *S* the point where their common ſection paſſes the plane of the figure, to which it is alſo perpendicular‖. Let *Q* be an object placed between the ſpeculums, and in the plane of the figure. Several repreſentations of that object will be formed by the ſucceſſive reflections of theſe ſpeculums; all which will lie in a plane paſſing through the object, and perpendicular to both the reflecting planes; that is in the plane of the figure as was before ſhewn, par. 2.§. But theſe repreſentations or images of the object *Q* will alſo lie in the circumference of a circle drawn on that plane, whoſe center is *S*, and radius *SQ* the perpendicular diſtance of the object from the common ſection of the two ſpeculums.

5. Draw *QT* perpendicular to the ſpeculum *SH*, produce *QT* to *A*, ſo that *TA*=*TQ*, and *Q* being conſidered as the focus of rays incident on *SH*, *A* will be the focus of re-

* Def. 6. Prop. 19. El. XI † 32 El. I.
‡ *Smith*'s Optics, Art. 9, &c. as before. ‖ Def. 6. and Prop. 19 El. XI. § *Smith*'s Optics, Art. 7, &c.

flected

flected rays*, or place of the image after one reflection from *SH*†. But the right angled triangles *QTS* and *ATS* having the sides *TA* and *TQ* equal, and *ST* common, will have their hypothenuses *SQ* and *SA* equal‡. Therefore the circle described with the center *S*, and radius *SQ* will pass through *A*.—Moreover the arch *HA* is equal to *HQ*.

6. For the like reason if *a* be the image made by one reflection from the other speculum *SI*, it will lie in the circumference of this circle passing through *Q*; and the arch *Ia* is equal to *IQ*.

7. The image *a* may be considered as an object in respect of the speculum *SH*‖. Take therefore the arch *HB* = *Ha*. and *B* will be the image of that image, or the image seen in the speculum *SH* by two reflections; first from *SI*, and then from *SH*§.

8. The image *A* may in like manner be considered as an object in respect of the speculum *SI*¶. Take the arch *Ib* = *IA*, and *b* is the other image made by two reflections and seen in the speculum *SI*.

9. Again take *HC* = *Hb*, and *C* will be the image seen in the speculum *SH* after three reflections; first from *SH*, then from *SI*, and then from *SH* again.

10. Take *Ic* = *IB*, and *c* is the other image seen (in *SI*) after three reflections.

11. Again take *ID* = *Hc*, and *D* is the image seen in *SH* after four reflections.

Take *Id* = *IC*, and *d* is the other image seen after four reflections.

12. Again take *HE* = *Hd*, and *E* is the image seen in *SH* after five reflections; and so on.

13. *Cor.* 1. Since *Qa* = 2*Ia*, and *Ba* = 2*Ha*, subtract the former from the latter, and there remains $\overline{QB = 2 \times Ha - Ia} = 2HI$. Again, *Bc* = 2*Ic* and *Dc* = 2*Hc*; subtract the former from the latter, and there remains $\overline{BD = 2 \times Hc - Ic} = 2HI$, and so on. Make therefore the arches *QB*, *BD*, *DF*. &c. each equal to 2*HI*; and *B*, *D*, *F*, &c.

* *Smith*'s Optics, Art. 202. † Ibid. Art. 25. ‡ 4 El. 1.
‖ *Smith*'s Optics, Art. 111. § Par. 5. ¶ *Smith*'s Optics, Art. 111.

will be the ſucceſſive images of the Object Q after an even number of reflections.

14. In like manner take Qb, bd, df, each equal to $2HI$; and b, d, f, &c. will be the place of the images ſeen in the other ſpeculum after an even number of reflections.

15. *Cor.* 2. The diſtances of each of the images B, D, F, &c. or of b, d, f, &c. from the Object Q, meaſured along the arch of the circle; is equal to arch HI multiplied by the number of reflections.

16. *Cor.* 3. Again, ſince $HA=HQ$, and $HC=Hb$, take the former from the latter, and there remains $AC=Qb=2HI$*. Therefore if QA be taken equal to $2QH$, and the ſeveral arches AC, CE, &c. each equal to $2HI$; then A, C, E, &c. will be the place of the images ſeen in SH after an odd number of reflections.

17. In like manner take $Qa=2QI$, and then the ſeveral arches ac, ce, &c each equal to $2HI$, and a, c, e, will be the place of the images ſeen in the other ſpeculum after an odd number of reflections.

18. *Cor.* 4. The diſtance of the firſt image from the object, or QA, is equal to $2QH$: that of the third image or QC $(=QA+AC)$ is equal to $2QH+2HI$; that of the fifth image (or $QA+AE$) is $2QH+4HI$, &c. the diſtance of each image from the object Q, meaſured along the arch of the circle, being equal to the arch HI multiplied by the number of reflections, ſave one, and increaſed by twice QH the diſtance of the object from the ſpeculum.

19. The ſame is true of a, c, e, &c. the images ſeen in the other ſpeculum after an odd number of reflections.

20. *Cor.* 5. If the two ſpeculums, keeping their inclination to each other, be turned round their common ſection as an axis; the images B, D, F, &c. formed by an even number of reflections will be immoveable; becauſe the intercepted arch HI is not altered: but the alternate images A, C, E, &c. formed by an odd number of reflections, will change their places by double the change of the arch QH, or double the angular motion of the ſpeculums.

* Par. 14.

21. *Cor.*

21. *Cor.* 6. Hence (in fig. 1) with the center *S* and radius *SB* deſcribe a circle cutting in *A*, the laſt reflected ray *HO* produced backwards; and *A* will be the place of the image of the object *B* after both reflections. For all the images of this object lie in the circumference of that circle*, and the laſt image muſt lie in the direction of the laſt reflected ray†; therefore it is in *A*, their common interſection.

22. Let an object be placed at *C* in the reflected ray *OH* produced, ſo as to be ſeen by direct viſion through the ſpeculum *H* along with the reflected image *A*. The image *A* will continue immoveable notwithſtanding the two ſpeculums be turned together round their common interſection as an axis‡; and therefore will ſeem always to adhere to the object *C*.

23. Let the eye be placed at *E*, the interſection of the incident ray with the ray after two reflections; then *BEA*, or *BEC*, the angle which theſe two rays make with each other, is alſo the angle which the object *B* makes with its image *A*: it is alſo the angle which the two objects *B* and *C* ſubtend to the naked eye at *E*. This angle *BEA* has been ſhewn to be equal to twice *ISH*‖.

24. If the eye be placed any where elſe in *IE* produced as at *O*, the doubly reflected image *A* and the object *C* will ſtill appear to coincide as before: but the angle which the objects *B* and *C* ſubtend to the naked eye is now *BOC*, differing from *BEA* by the angle *EBO*, or *IBO*§.

25. Let now the ſpeculum *NIn* be turned about the point *I*, ſo that the doubly reflected image of the object *C* (placed in the line *HO* produced) may be made to *coincide* with that object itſelf ſeen directly by the eye at *O*; then is the angle *ICH* double of *ISH*, the angle between the ſpeculums. For in this caſe the incident ray is *CI*, and the ray after two reflections is *HO*, which produced backwards paſſes through *C* by the ſuppoſition of the coincidence aforeſaid. Therefore the interſections of theſe two rays is at *C* the object itſelf, and conſequently *ICH*, the angle of their interſection, is double of *ISH* the angle of the ſpeculums.¶

* Par. and following. † *Smith*'s Optics, Art. 101, 102.
‡ Par. 20. ‖ Ibid. 2. § 32 El. 1. ¶ Par. 2.

26. The interſection, before marked *E*, in this caſe removes to *C* on the other ſide of *H*; and *S* the point of interſection of the two ſections *NIn* and *RHr*, falls on the other ſide of the line *IH*.*

27. If the object *C* be removed to ſuch a diſtance that the angle *ICH* vaniſhes, the angle *ISH* will alſo vaniſh, and and the ſpeculums become parallel to each other.

* To conſider the motion of the points *E* and *S* more particularly, imagine the ſpeculum *NIn* to turn round the point *I*, while the other ſpeculum *RHr*, and likewiſe the poſition of the ray *HO*, remains fixed.

And firſt let the ſpeculum *NIn* turn ſo that the point *S* may continually recede from *H*. Then the point *E* will continually recede from *H* towards *O* When the two ſpeculums become parallel, the point *S* and likewiſe the point *E* will go off *in infinitum*, as is evident, becauſe *BEA* is twice *ISH*; therefore when the latter vaniſhes, ſo dues the former. In this caſe the incident ray and ray after two reflections are parallel.

Continue the motion of the ſpeculum *NIn*, and the ſections *NIn* and *RHr* will now meet on the other ſide, in *SH* continued. The interſection of the incident ray and ray after two reflections will now alſo remove to the other ſide, in *OH* continued. This will appear, if inſtead of conſidering *HO* as the ray after two reflections, we conſider it as the incident ray, and therefore *HI* as a ray given in poſition and incident on the ſecond ſpeculum *NIn*; *IC* will then be the ray after two reflections by *Smith*'s Optics, Art. 11.

Continue the motion of the ſpeculum *NIn*, and the point of interſection of theſe two rays will continue its motion from *A* towards *H*, and paſſing the point *H* will recede beyond *O* *in infinitum*. Thus theſe two rays will again be parallel. This happens when the ſpeculum *NIn* makes a right angle with the ſpeculum *RHr*.

Continue this motion, and the interſection of the incident ray and ray after two reflections will a ſecond time remove to the other ſide, in *OH* continued. When the face of the ſpeculum comes into the direction *IH*, the interſection of thoſe two rays will then be in *H*. After this the back of the ſpeculum will be turned towards *RHr* for an intire ſemi-revolution, till the face again comes into the direction *IH*, and the two rays aforeſaid again interſect in *H*; the ſpeculums then making an acute angle the ſame way as at firſt.

If the motion of the ſpeculums be ſtill continued, the interſection of the two rays at *E* will recede from *H* towards *O* *in infinitum*, till the ſpeculums are parallel as before.

The incident ray and ray after two reflections are twice parallel in one ſemi-revolution of the ſpeculum *NIn*; *viz.* when the two ſpeculums are parallel, and alſo when they are perpendicular to each other. In the other ſemi-revolution, the back of the ſpeculum *NIn* is turned towards the other ſpeculum *RHr*.

The

The incident ray *CI*, and the ray after two reflections *HO*, in this caſe are alſo parallel, their interſection *C* being removed to an infinite diſtance.

28. *Scholium.* If when the index ſtands at the beginning of the diviſions, the two ſpeculums are put into a parallel poſition, by turning one of them about till the reflected image of ſome diſtant object *C* coincides with that object ſeen directly, the inſtrument is then ſaid to be *adjuſted*. The inſtrumeut being thus adjuſted and the ſpeculums fixed, the index will always ſhew the double inclination of the ſpeculums, either on the quadrantal arch, or arch of exceſs, according as the point *S* falls on one ſide or the other of the perpendicular let fall from *I* on *RHr* produced.

If, when the ſpeculums are put into a parallel poſition as before, the index points to any number of degrees or minutes on the quadrantal arch (inſtead of 0) then it will always give the double inclination too much on the quadrantal arch, and equally too little on the arch of exceſs, by the number of degrees and minutes to which the index thus points, and contrariwiſe.

29. The angle *BIC*, which any two objects *B* and *C* ſubtend at *I* the center of the index glaſs, is equal to the ſum of the two angles *BEC* and *ICE*.*

30. *Scholium* 2. The quadrant being adjuſted as before by a diſtant object, the former of theſe angles *BEC* will be ſhewn on the quadrantal arch when the reflected image of *B* coincides with *C* ſeen directly. The latter of theſe angles *ICE* will be ſhewn on the arch of exceſs, when the reflected image of *C* coincides with that object itſelf ſeen directly. Or the ſum of theſe two angles will be ſhewn at once on the quadrantal arch, if, when the object *C* and its image coincide, the index be made to point at the beginning of the diviſions; and then be carried forward till the reflected image of *B* coincides with *C* ſeen directly.

31. In figure 4. let *I* be the index-glaſs, ſtanding parallel to *H* the fore-horizon glaſs; let *R* be the back hori-

* 32 El. I.

zon

zon glaſs, ſtanding at right angles to *H* and *I*. Draw *PI* and *PR* perpendiculars to *I* and *R* meeting each other in *P*. Let *QH* be perpendicular to *H*. Let *BI* be the ray incident on *I*, and reflected from thence into *IH*, from thence into *HO*. the axis of viſion for the fore horizon glaſs. Let *IR* be the ray reflected from the index-glaſs to the back horizon-glaſs, and from thence into *RY* the axis of viſion for that glaſs. Then by the laws of reflection $PIH = PIB$.* Moreover $PIH = IHQ$.† Therefore the angle which the line, joining the centers of the index-glaſs and fore horizen-glaſs, makes with a perpendicular to the index-glaſs, when the index ſtands at the beginning of the diviſions, is equal to the angle of incidence on the index-glaſs in that poſition; and this is the conſtant angle of incidence on the fore horrizon-glaſs.

32. That $PIR = PIH + HIR$ is ſelf-evident, whatever be the poſition of the perpendicular *PI*. But *PIH* is the angle of reflection from the index-glaſs to the fore horizon-glaſs, and *PIR* the angle of reflection to the back horizon-glaſs, which are equal to their reſpective angles of incidence. Therefore the angle of incidence on the index-glaſs in the back obſervation, exceeds the angle of incidence in the fore obſervation (the perpendicular *PI* ſtanding in the ſame place) by the angle *HIR* which the centers of the two leſs mirrors ſubtend at the center of the great one.

33. When *I* and *H* are parallel, *PIH* is equal to *IHQ*, and *IPR* is a right angle. Therefore *PRI* is the complement of *PIR*, or of $PIH + HIR$, that is of $IHQ + HIR$. Therefore the angle of incidence on the back horizon-glaſs, is conſtantly the complement to 90 degrees of the ſum of the angle of incidence on the fore horizon glaſs, and the angle which the centers of the two leſs mirrors ſubtend at the center of the great one.

34. Let *RN* repreſent the ſurface of the back horizon-glaſs, *RY* the ray reflected from it or the axis of viſion, and produce *YR* to *M*. Let *OH* be the axis of viſion for the fore obſervation. Produce *OH* till it meets *RY* in *Y*,

* *Smith*'s Optics, Art. 8. † 29 El. I.

and

and let *OH* meet *IR* in *Z*. Then the angle *RYZ*, which the two axes of viſion make with each other, is the difference of *ZRM* and *YZR*.* Now the former of theſe $ZRM = 2 \times ZRN$† $= 2 \times PIR$‡ $= 2 \times PIH + 2 + IIIZ$. The latter of thoſe angles or $YZR = ZHI + HIZ$* $= BIH + HIZ$‡ $= 2 \times PIH + HIZ$.† Subtract the latter from the former, and we have their difference, or *RYZ* equal to *HIZ*, or *HIR*. Therefore the angle which the two axes of viſion (for the fore and back obſervation) make with each other is equal to that which the two leſs mirrors ſubtend at the center of the great one.

35. To ſhew the correction neceſſary to be made in taking the height of a building by reflection from water, let figure 5 repreſent the poſition of the octant when the fore obſervation is uſed; figures 6 and 7 the poſition of the octant in the back obſervation. In theſe figures let *LR* be the ſurface of the water. *BL* the building, *AL* its reflected image ſeen in the water. *R* the point of reflection from the water. *I* the point of reflection from the index-glaſs. *H* the point of reflection from the horizon-glaſs, through which the extremity of the image *A* is ſeen in the water. *O* the place of the eye. *E* the interſection of the ray (from *B* the top of the building) incident on the index-glaſs with that ray after two reflections. *S* the point where the ſections of the ſpeculums by the plane of the octant meet each other; and continue *HS* to *T* in figures 6 and 7. Then is *AEB* the angle ſhewn on the limb, being double of *ISH* in figure 5, and double of *IST* in figure 6 and 7,‖ and *BRL* is the angle of the elevation of the building above the level of the water, ſuppoſing the eye placed at *R* the point of reflection. Now $2 \times BRL = BRA$† $= AEB$ *EBR* in figures 5 and 6, and equal to *AEB*—*EBR* in Figure 7*; that is $2 \times BRL = AEB + IBR$ in figures 5 and 6, and equal to *AEB*—*IBR* in figure 7. Eſtimate therefore the angle *IBR*, which the middle of the index-glaſs, and the point of reflection from the water, makes at the top of the building: add this to the Angle ſhewn on the limb in the fore obſervation; add

* 32 El. I. † *Smith*'s Optics. Art. 8. ‡ 29 *El. I.*

‖ Par. 1, &c.

or

or ſubtract it in the back obſervation, according as the ray incident on the water paſſes without or within the index-glaſs; and the ſum or difference gives the angle *BRA*; half of which is the true angle of elevation.

36. To find the errors occaſioned by a given deviation of the reflecting planes from their perpendicular poſition.

It may be convenient to conſider the planes in queſtion as the planes of ſeveral great circles of a ſphere; and the point in which the two reflecting planes, and plane of the octant all concur, as the center of that ſphere. Their mutual inclinations may then be found by the common rules of ſpherical trigonometry.

37. In figure 8 let *C* be the center of the ſphere, where the three planes before mentioned all meet. Let the plane of the great circle *TNEH* repreſent the plane of the octant. Let the plane of the great circle *NCP* repreſent the plane of the index-glaſs; and the plane of the great circle *HCP*, the plane of the horizon-glaſs. Then is *NC* and *HC* the interſections of the two reflecting planes with the plane of the octant, and *PC* their common interſection with each other. Moreover the ſpherical angles *PNT* and *PHN* are the inclinations of the reflecting planes to the plane of the octant which we here ſuppoſe equal; the ſpherical angle *NPH* is their inclination to each other. The arch *NH* meaſures the angle *NCH*, which the two ſections of the reflecting planes by the plane of the octant, make with each other; and which is half the angle ſhewn by the index on the limb. With the pole *P* deſcribe a great circle *LDEF*, cutting the circle *PND* at right angles in *D*; the circle *PFH* at right angles in *F*, and the circle *TNEH* obliquely in *E*; *CE* being the interſection of the plane of the oblique circle *LDEF* with the plane of the circle *TNEH*. Through *P* and *E* draw a great circle; this will meet the circle *LDEF* at right angles in *E*; and the plane of this circle will paſs through both *PC*, the common interſection of the reflecting planes, and alſo through *CE* the interſection of the plane of the oblique circle *LDEF* with the plane of the circle *TNEH*; the plane of this circle is ſeen edgewiſe in the figure. The ſpherical angle *PEN* which this circle makes with the circle *TNE* will be the inclination of *PC* the common interſection

terſection of the reflecting planes to the plane of the circle *TNEH*, that is the plane of the octant, and *NED* will be the complement of this angle. Laſtly, let the radius *TC*, lying in the plane of the great circle *TNEH*, repreſent a line drawn on the plane of the octant parallel to the axis of the telescope; paſſing both through *CH* the common ſection of the plane of the octant and horizon-glaſs, and through *PC* the common ſection of the reflecting planes. Through *P* and *T* draw the great circle *PTL*, meeting the circle *TNE* obliquely in *T*, but the circle *LDEF* at right angles in *L*; then will *TCH* be the (obtuſe) angle which the line *TC* makes with the ſection *CH*. The arch *PT* is the meaſure of *PCT*, the angle which the line *TC* makes with *PC*, the common ſection of the reflecting planes; the arch *TL* is the complement of the arch *PT*, or it is the diſtance of the parallel circle from the great circle, mentioned by Hadley in his fifth corollary; that great circle being here repreſented by *LDEF**.

38. Fig. 9. repreſents the ſtereographic projection of theſe ſeveral circles on the plane of the circle *TNEH* in figure 8, that is, on the plane of the octant; the dotted lines *CT*, *CN*, *CH*, repreſent the ſeveral radii *CT*, *CN*, *CH*, lying in the plane of the circle *TNEH*. The ſeveral circles of the ſphere before mentioned are all repreſented by circles in figure 9, according to the laws of the ſtereographic projection.

39. Theſe things laid down, becauſe the ſpherical angles *PNT* and *PHN* are by ſuppoſition equal, the arches *EN* and *EH*, likewiſe *ED* and *EF*, are reſpectively equal: whence *EN* is equal to $\frac{1}{2}$ *NH*; therefore *EN* is the meaſure of $\frac{1}{4}$ of the angle ſhewn by the index on the limb. The arch $ED = \frac{1}{2} DF$, therefore *ED* is the meaſure of half the ſpherical angle *NPH*, or of half the true inclination of the reflecting planes to each other, or it is the meaſure of $\frac{1}{4}$ of the corrected angle. Now in the right angled ſpherical triangle *NDE* we have the hypothenuſe *EN*, and the angle $END = PNT$, to find the baſe *ED*. Therefore as radius is to ſine *END* (or coſine of deviation from the

* See Philoſ. Tranſ. Nr. 420.

perpendicular;) ſo is ſine *EN* to ſine *ED*, which multiplied by 4 gives the corrected angle.

40. Next to find the inclination of *PC* the common ſection of the reflecting planes to the plane of the octant, or to find *NED* the complement of that angle. In the right-angled triangle *NDE* we have the hypothenuſe *EN* and the angle *END* as before, to find the other angle *NED*. Therefore as radius is to the tangent *END*, ſo is coſine *EN* to cotangent *NED*, or the tangent of the inclination of the common ſection of the reflecting planes to the plane of the octant.

41. Next to find *PCT* the angle which a line drawn on the plane of the octant, parallel to the axis of the teleſcope and paſſing through the common ſection of the reflecting planes, makes with that common ſection; or rather to find *TL* the arch which meaſures the complement of that angle. Firſt, we have *ET*=*TH*—*HE*=*TH*—*EN*. Then in the right-angled ſpherical triangle *TLE* we have the hopothenuſe *ET*, the angle *TEL*=*NED* (juſt found) to find the perpendicular *TL*. Therefore as radius is to ſine *ET*, ſo is ſine *TEL* to ſine *TL*, or diſtance of the parallel from the great circle in Hadley's fifth corollary.

42. Laſtly, by that corollary, as radius is to the ſine complement of that diſtance, ſo is the chord of double the true inclination of the reflecting planes, or chord of the corrected angle, to the chord of the true angle between the obſerved objects: that is, radius is to the ſine complement of the diſtance aforeſaid, as the ſine of half the corrected angle to the ſine of half the true angle between the objects; the double of which is the angle required.

Otherwiſe having obtained the corrected angle and the diſtance of the parallel from the great circle, ſay, as radius is to the tangent of half the corrected angle, ſo is double the verſed ſine of that diſtance*, to the ſine of an angle which, being ſubtracted from the corrected angle, gives the true angle between the objects. This rule follows immediately from that given by Hadley in the latter part of his fifth corollary.

* Double the verſed ſine of any arch is a third proportional to the radius and chord of that arch.

43.

43. Of the effect of the refractions of the rays in a glaſs mirror whoſe two ſurfaces are parallel.

Let *AL* and *BN*, in figure 10, be the two parallel ſurfaces of a glaſs mirror; *Q* the focus of incident rays; *QC* a ray incident on the firſt ſurface at *C*: through *Q* draw *QABS* perpendicular to both ſurfaces, cutting the firſt ſurface in *A*, the ſecond in *B*, produce *AQ* backwards to *R*, ſo that *AR* may be to *AQ* as the ſine of incidence to the ſine of refraction, or as *m* to *n*, and *R* will be the focus after refraction at *C**: draw *CD* the refracted ray meeting the ſecond ſurface in *D*: in the perpendicular *RS*, take *BS* equal to *BR*, and *S* will be the focus of rays reflected from the ſecond ſurface at *D*†: draw *DE* the reflected ray meeting the firſt ſurface in *E*: take *AT* to *AS* as *n* to *m*, and *T* will be the focus of emergent rays after their laſt refraction at *E**; that is, *T* will be the place of the laſt image formed by two refractions and one reflection: take *AV*, equal to *AQ*, and *V* will be the place of the image formed by the reflection of the firſt ſurface only, and *TV* will be the diſtance between theſe two images: I ſay that *TV* will be to *AB*, the thickneſs of the glaſs, as twice the ſine of refraction to the ſine of incidence.

44. Produce *TE* to *M*, ſo that *EM* may be the emergent ray: through *D* draw the perpendicular *DH*, and through *E* and *C* the perpendiculars *be* and *dc*. Produce *QC* till it meets *TE* in *F*, and through *F* draw *FG* perpendicular to *QT*. Now $dCD = CDH$‡ $= HDE$§ $= DEb$‡; but *DC* and *DE* may be conſidered as rays incident on *AL*¶; and, becauſe the angles of incidence *dCD* and *DEb* are equal, therefore their angles of refraction *cCQ* and *eEM* are equal; but $cCQ = GQF$‡, and $eEM = GTF$‡, therefore the triangle *QFT* is iſoceles, and $QG = GT$. Now $AT = GT + AG$ and $AV = AQ = GQ - AG = GT - AG$; whence $AT - AV$ or $TV = GT + AG - GT - AG = 2 \times AG$. Now the triangles *CFH* and *CQA*, as alſo *CDH* and *CRA*, are reſpectively ſimilar: hence *HF* is to *AQ*, alſo *HD* to *AR*, as *CH* to *CA*; therefore *HF* is to *HD* as *AQ* to *AR*‖; that

* *Smith*'s Optics, Art. 223. † Ibid. Art. 202. ‡ 29 El. I.
§ *Smith*'s Optic's, Art. 8. ¶ Ibid. Art. 11. ‖ 11 and 16 El. V.

is, AG is to AB as AQ to AR, or as n to m; consequently $2\times AG$ or TV is to AB as $2n$ to m.

45. *Cor.* 1. If m be to n as 3 to 2, then $TV=\frac{4}{3}AB$, and $AG=\frac{2}{3}AB$, and $EG=\frac{1}{3}AB$.

46. *Cor.* 2. The laſt emergent ray proceeds exactly as if the reflection had been made without refraction, by a plane FG parallel to BN the back ſurface, and diſtant from it $\frac{1}{3}$ of the thickneſs of the glaſs.

47. *Scholium.* The index-glaſs ſhould be ſo fixed on the index, that the center on which the index turns may paſs through the line FG; that is, be diſtant from the back ſurface $\frac{1}{3}$ of the thickneſs of the glaſs, and be within it.

48. We ſhall here juſt mention the conſequences that would follow from ſuppoſing the two ſurfaces of each glaſs mirror inclined to one another in a given angle; and we will conſider, firſt, the effect of the refractions of the index-glaſs only.

49 When there is no refraction, or when the two ſurfaces of the index-glaſs are parallel, then the laſt image is removed from the object by twice the inclination of the reflecting ſurfaces of the two ſpeculums, as we have ſhewn before. But, if the outer ſurface of the index-glaſs is inclined to the inner ſurface, the place of that image will then be changed by the refractions at the outer ſurface of the index-glaſs. The laſt image will, in this caſe, be formed by two refractions and one reflection at the index-glaſs: and its angular diſtance from the object will be increaſed, if the refracting angle of the index-glaſs be turned the ſame way with the angle which the ſpeculums make with each other; and decreaſed if the refracting angle be turned the contrary way. This angular change in the place of the laſt image on account of refractions we will call *the angle of deviation.*

50. Now, was the angle of deviation always the ſame in every poſition of the index-glaſs, the refractions then would occaſion no error in the obſerved angle. For in this caſe the image, formed both by refraction and reflection, keeping one conſtant diſtance from that which would be formed by reflection only, will have the ſame angular motion, *viz.* double that of the index-glaſs. Therefore if

any

any diſtant object and its laſt image, thus formed, be made to coincide when the index ſtands at the beginning of the diviſions, the index will then always ſhew the angular diſtance of this laſt image from the object, and, conſequently, the angular diſtance of any other object, which is ſeen *along with* that laſt image; juſt as when there is no refraction. All the difference will be, that the reflecting ſurface of the index-glaſs will not ſtand exactly parallel to the horizon-glaſs when the object and it's image are made to coincide, in order to adjuſt the inſtrument.

51. When the rays fall ſo nearly perpendicular on the index-glaſs, that the angles of incidence and refraction may be accounted to have the given ratio of their ſines; then the angle of deviation is to the angle of inclination of the two ſurfaces of the index-glaſs, as double the difference between the ſines of incidence and refraction is to the leſs of thoſe ſines. Hence the angle of deviation is nearly equal to the angle of the inclination of thoſe ſurfaces.*

52. But, as thoſe angles are not accurately as their ſines, ſo the angle of deviation is not exactly what that rule gives it; nor the ſame in all poſitions of the index-glaſs.† It is always greater than what the rule gives; but it is leaſt of all when the ray within the glaſs falls perpendicularly on the reflecting ſurface, ſo that the firſt incident and laſt emergent rays coincide. This will be the caſe when *the index is in ſuch a poſition, that the horizon-glaſs may be ſeen by it's own reflection.* The incident ray in this caſe falls obliquely on the outer ſurface of the index-glaſs, and on that ſide of it's perpendicular which is contrary to the refracting angle. We ſhall call this ray, *the ray of leaſt deviation.*

53.

* In the Philoſophical Tranſactions, for 1772, vol. lxii, page 114, a different rule is given for finding the deviation: But, by *deviation* is there meant a different thing, namely, the difference in the place of the image, when formed by two refractions and one reflection, as before; and the image by one reflection, not at the back or ſilvered ſurface, as ſuppoſed in par. 49, but at the fore or unſilvered ſurface.

† For this reaſon it is propoſed, in the Philoſophical Tranſactions aforeſaid, that, in obſervations of the ſun or moon, the reflection ſhould be made from the fore ſurface only; the back ſurface being made rough and black.

53. The position of the index, when the deviation is leaſt, being determined as before, the deviation will continually increaſe, as the diſtance of the index from this poſition increaſes on either ſide. If equal angular diſtances of the index from this poſition be taken on contrary ſides, the angles of deviation in theſe two poſitions of the index will be nearly equal to each other, but not exactly ſo. That poſition of the index, in which the ray incident on the outer ſurface falls on the ſame ſide of the ray of leaſt deviation with the perpendicular to that ſurface, will have the greateſt angle of deviation.

54. All theſe conſequences follow from the common properties of the priſm, and need not be here particularly demonſtrated. If a priſm, whoſe refracting angle is about ten degrees, be properly placed before an index glaſs whoſe ſurfaces are parallel, all theſe conſequences may be ſhewn experimentally.*

55. We have hitherto taken no notice of the refractions of the horizon glaſs: it is becauſe theſe refractions can never affect the obſerved angle; for, as the horizon glaſs is fixed, the rays always fall upon it with the ſame degree of obliquity. Hence the change in the place of the image, on account of the refractions of this glaſs, is conſtantly the ſame. The refractions of this glaſs may change a little the poſition of the axis of viſion, but cannot alter the obſerved angle; not though the upper part of this glaſs be cut off, ſo that the object to be ſeen by direct viſion may be unrefracted,† while the refractions affect the other object.

* It may be neceſſary to meaſure practically the angle of the priſm. The following method is a very accurate one. Place the priſm in a vertical poſition. Then, ſtanding before the angle to be meaſured, place two candles or other proper objects, ſo that the image of one may be ſeen by reflection from one of the ſides, and the image of the other be ſeen by reflection from the other of the two ſides that form the refracting angle. Move theſe two objects till their reflected images coincide. Meaſure then the diſtances of theſe objects from each other and from the angle of the priſm; from theſe diſtances compute the angle which the two objects ſubtend at the priſm: half this is the angle ſought. It may be neceſſary to cover the back of the priſm, that the light refracted through the other two angles may not mix with that reflected from the ſides.

† The angle of deviation from refraction only is half that cauſed by refraction and reflection together, as before.

ject. This will only alter the position of the glasses in the adjustment, as before. In like manner, if the surfaces of the dark glasses be not parallel, their refractions will not affect the observations, provided the instrument be adjusted by the sun, or some object whose reflected image is seen through them.

56. If the inclination of the two surfaces of any of the glasses be considerable, the object will appear coloured, or at least indistinct; and, if the instrument be turned about, (in its own plane,) the place of the reflected image will now be changed; because the angle of incidence, and therefore the effect of the refractions upon that image, will be changed. The multiplicity of the images may also create confusion. Upon these accounts those surfaces ought to be made parallel: but, if this was not the case, we may see that it is impossible to remedy *perfectly* the other errors from refraction, notwithstanding what has been said by some persons of distinguished learning.*

* See the *American Transactions*, Vol. I. printed at *Philadelphia*, 1771, Appendix, page 21.

THE END.